

JEE (ADVANCED) 2019 PAPER 1

PART-I PHYSICS

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3 If ONLY the correct option is chosen;
Zero Marks	: 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -1 In all other cases.

Q.1 Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K . The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is
 [G is universal gravitational constant]

(A)
$$\frac{K}{2\pi r^2 m^2 G}$$

(B)
$$\frac{K}{\pi r^2 m^2 G}$$

(C)
$$\frac{3K}{\pi r^2 m^2 G}$$

(D)
$$\frac{K}{6\pi r^2 m^2 G}$$

Q.2 A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0 . A hole with a small area $\alpha 4\pi R^2$ ($\alpha \ll 1$) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct?

(A) The potential at the center of the shell is reduced by $2\alpha V_0$

(B) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(C) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(D) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance $2R$ from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

Q.3 A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

$$T(t) = T_0(1 + \beta t^{\frac{1}{4}}),$$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is,

(A) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$ (B) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (C) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$ (D) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

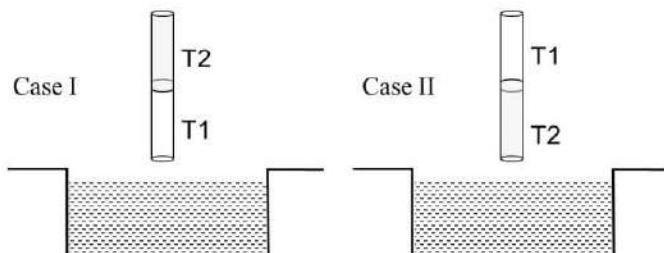
Q.4 In a radioactive sample, $^{40}_{19}K$ nuclei either decay into stable $^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei are produced by the $^{40}_{19}K$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei to the radioactive $^{40}_{19}K$ nuclei is 99, the value of t will be,
[Given: $\ln 10 = 2.3$]

(A) 1.15 (B) 9.2 (C) 2.3 (D) 4.6

SECTION 2 (Maximum Marks: 32)

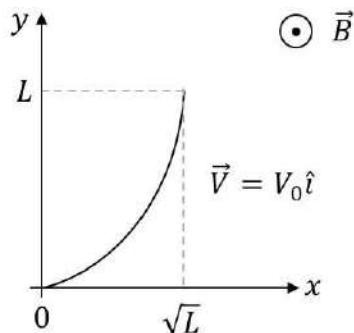
- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
 - Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
 - Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
 - Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
 - Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
 - Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
 - Negative Marks : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get -1 mark.

Q.5 A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T1 and T2 of different materials having water contact angles of 0° and 60° , respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct?
 [Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3 , take $g = 10 \text{ m/s}^2$]



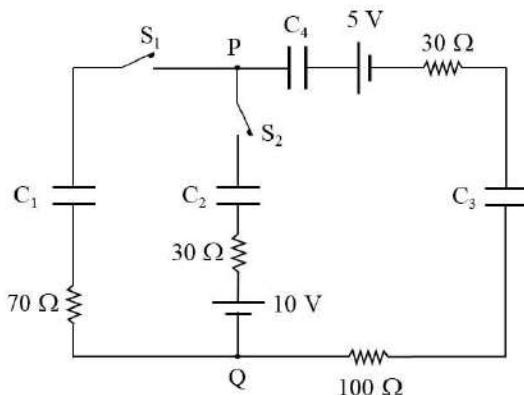
- (A) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.
- (B) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)
- (C) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)
- (D) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

Q.6 A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0 \hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0 \left(1 + \left(\frac{y}{L}\right)^\beta\right) \hat{k}$, as shown in figure. If V_0 , B_0 , L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:



- (A) $|\Delta\phi| = \frac{1}{2} B_0 V_0 L$ for $\beta = 0$
- (B) $|\Delta\phi| = \frac{4}{3} B_0 V_0 L$ for $\beta = 2$
- (C) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2} L$
- (D) $|\Delta\phi|$ is proportional to the length of the wire projected on the y -axis.

Q.7 In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10 \mu\text{F}$, $C_2 = 30 \mu\text{F}$ and $C_3 = C_4 = 80 \mu\text{F}$.



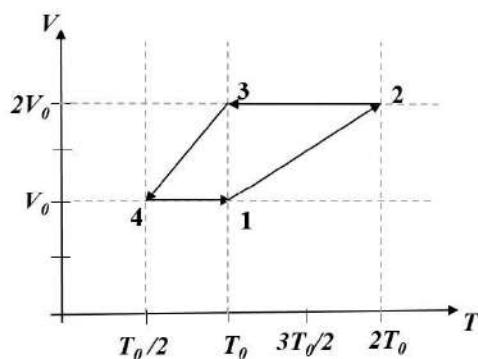
Which of the statement(s) is/are correct?

- (A) At time $t = 0$, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.
- (B) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be 4 V.
- (C) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).
- (D) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

Q.8 A charged shell of radius R carries a total charge Q . Given Φ as the flux of electric field through a closed cylindrical surface of height h , radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?
 $[\epsilon_0$ is the permittivity of free space]

- (A) If $h > 2R$ and $r > R$ then $\Phi = Q/\epsilon_0$
- (B) If $h < 8R/5$ and $r = 3R/5$ then $\Phi = 0$
- (C) If $h > 2R$ and $r = 3R/5$ then $\Phi = Q/5\epsilon_0$
- (D) If $h > 2R$ and $r = 4R/5$ then $\Phi = Q/5\epsilon_0$

Q.9 One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature ($V-T$) diagram. The correct statement(s) is/are:
 [R is the gas constant]



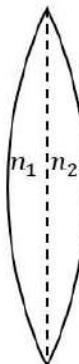
(A) Work done in this thermodynamic cycle ($1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$) is $|W| = \frac{1}{2}RT_0$

(B) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(C) The ratio of heat transfer during processes $1 \rightarrow 2$ and $2 \rightarrow 3$ is $\left| \frac{Q_{1 \rightarrow 2}}{Q_{2 \rightarrow 3}} \right| = \frac{5}{3}$

(D) The ratio of heat transfer during processes $1 \rightarrow 2$ and $3 \rightarrow 4$ is $\left| \frac{Q_{1 \rightarrow 2}}{Q_{3 \rightarrow 4}} \right| = \frac{1}{2}$

Q.10 A thin convex lens is made of two materials with refractive indices n_1 and n_2 , as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n \ll (n - 1)$ and $1 < n < 2$, the correct statement(s) is/are,



A) $\left| \frac{\Delta f}{f} \right| < \left| \frac{\Delta n}{n} \right|$

B) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

C) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

D) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

Q.11 Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L , which of the following statement(s) is/are correct?

(A) The dimension of linear momentum is L^{-1}

(B) The dimension of energy is L^{-2}

(C) The dimension of force is L^{-3}

(D) The dimension of power is L^{-5}

Q.12 Two identical moving coil galvanometers have 10Ω resistance and full scale deflection at $2 \mu\text{A}$ current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with $R = 1000 \Omega$ resistor by using an ideal cell. Which of the following statement(s) is/are correct?

(A) The resistance of the Voltmeter will be $100 \text{ k}\Omega$

(B) The resistance of the Ammeter will be 0.02Ω (round off to 2nd decimal place)

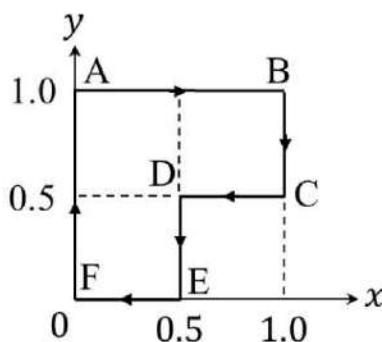
(C) The measured value of R will be $978 \Omega < R < 982 \Omega$

(D) If the ideal cell is replaced by a cell having internal resistance of 5Ω then the measured value of R will be more than 1000Ω

SECTION 3 (Maximum Marks: 18)

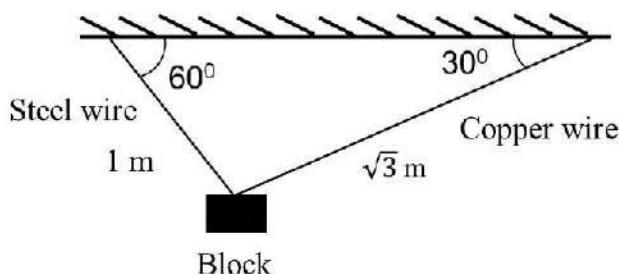
- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +3 If ONLY the correct numerical value is entered;
 Zero Marks : 0 In all other cases.

Q.13 A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y \hat{i} + 2\alpha x \hat{j})$ N, where x and y are in meter and $\alpha = -1 \text{ Nm}^{-1}$. The work done on the particle by this force \vec{F} will be ____ Joule.

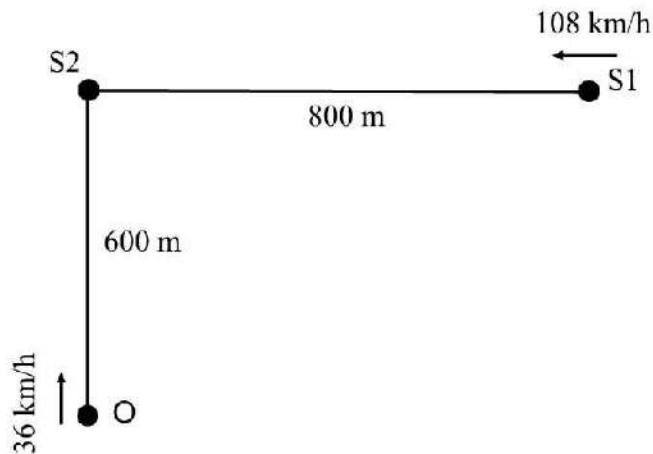


Q.14 A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3} \text{ m}$ and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60° , respectively. If elongation in copper wire is (Δl_C) and elongation in steel wire is (Δl_S) , then the ratio $\frac{\Delta l_C}{\Delta l_S}$ is ____.

[Young's modulus for copper and steel are $1 \times 10^{11} \text{ N/m}^2$ and $2 \times 10^{11} \text{ N/m}^2$, respectively.]



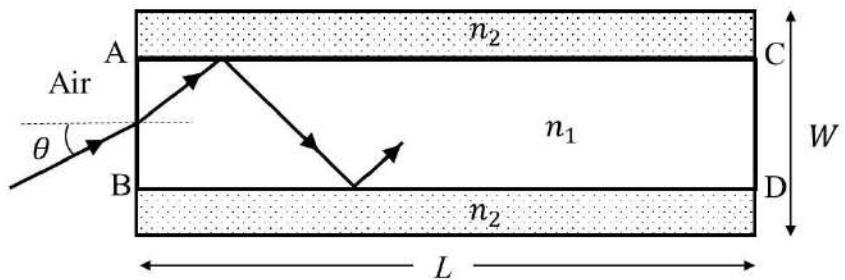
Q.15 A train S₁, moving with a uniform velocity of 108 km/h, approaches another train S₂ standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S₂, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S₂ and distance between S₁ and S₂ is 800 m, the number of beats heard by O is _____.
[Speed of the sound = 330 m/s]



Q.16 A parallel plate capacitor of capacitance C has spacing d between two plates having area A . The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K \left(1 + \frac{m}{N}\right)$. For a very large N ($> 10^3$), the capacitance C is $\alpha \left(\frac{K\epsilon_0 A}{d \ln 2}\right)$. The value of α will be _____.
 $[\epsilon_0$ is the permittivity of free space]

Q.17 A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ____ °C.
[Neglect the heat exchange with surrounding]

Q.18 A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L \gg W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For $L = 9.6$ m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is _____. [Speed of light $c = 3 \times 10^8$ m/s]



JEE (ADVANCED) 2019 PAPER 1
 PART-III CHEMISTRY

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

<i>Full Marks</i>	: +3	If ONLY the correct option is chosen;
<i>Zero Marks</i>	: 0	If none of the options is chosen (i.e. the question is unanswered);
<i>Negative Marks</i>	: -1	In all other cases.

Q.1 The green colour produced in the borax bead test of a chromium(III) salt is due to

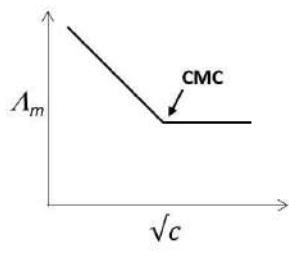
(A) $\text{Cr}(\text{BO}_2)_3$ (B) $\text{Cr}_2(\text{B}_4\text{O}_7)_3$ (C) Cr_2O_3 (D) CrB

Q.2 Calamine, malachite, magnetite and cryolite, respectively, are

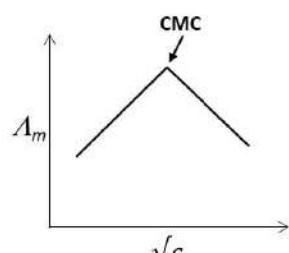
(A) ZnSO_4 , CuCO_3 , Fe_2O_3 , AlF_3
 (B) ZnSO_4 , $\text{Cu}(\text{OH})_2$, Fe_3O_4 , Na_3AlF_6
 (C) ZnCO_3 , $\text{CuCO}_3 \cdot \text{Cu}(\text{OH})_2$, Fe_3O_4 , Na_3AlF_6
 (D) ZnCO_3 , CuCO_3 , Fe_2O_3 , Na_3AlF_6

Q.3 Molar conductivity (A_m) of aqueous solution of sodium stearate, which behaves as a strong electrolyte, is recorded at varying concentrations (c) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution? (critical micelle concentration (CMC) is marked with an arrow in the figures)

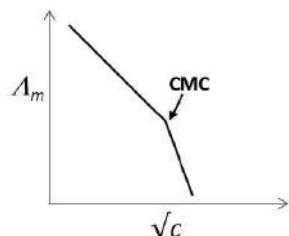
(A)



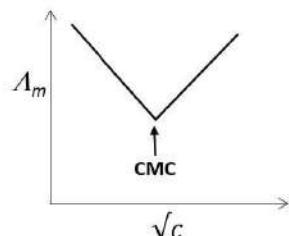
(B)



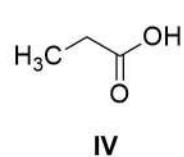
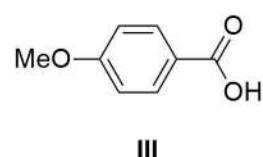
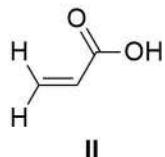
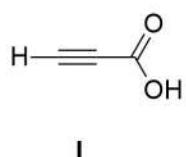
(C)



(D)



Q.4 The correct order of acid strength of the following carboxylic acids is



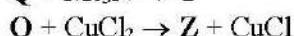
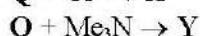
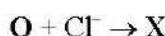
- (A) III > II > I > IV
- (B) I > II > III > IV
- (C) I > III > II > IV
- (D) II > I > IV > III

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -1	In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
 - choosing **ONLY** (A), (B) and (D) will get +4 marks;
 - choosing **ONLY** (A) and (B) will get +2 marks;
 - choosing **ONLY** (A) and (D) will get +2 marks;
 - choosing **ONLY** (B) and (D) will get +2 marks;
 - choosing **ONLY** (A) will get +1 mark;
 - choosing **ONLY** (B) will get +1 mark;
 - choosing **ONLY** (D) will get +1 mark;
 - choosing no option (i.e. the question is unanswered) will get 0 marks; and
 - choosing any other combination of options will get -1 mark.

Q.5 A tin chloride **Q** undergoes the following reactions (not balanced)



X is a monoanion having pyramidal geometry. Both **Y** and **Z** are neutral compounds.
 Choose the correct option(s)

- (A) The central atom in **X** is sp^3 hybridized
- (B) There is a coordinate bond in **Y**
- (C) The oxidation state of the central atom in **Z** is +2
- (D) The central atom in **Z** has one lone pair of electrons

Q.6 Fusion of MnO_2 with KOH in presence of O_2 produces a salt W. Alkaline solution of W upon electrolytic oxidation yields another salt X. The manganese containing ions present in W and X, respectively, are Y and Z. Correct statement(s) is(are)

(A) In aqueous acidic solution, Y undergoes disproportionation reaction to give Z and MnO_2
(B) Both Y and Z are coloured and have tetrahedral shape
(C) Y is diamagnetic in nature while Z is paramagnetic
(D) In both Y and Z, π -bonding occurs between *p*-orbitals of oxygen and *d*-orbitals of manganese

Q.7 Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation.

(A) $2\text{H}_2(\text{g}) + \text{O}_2(\text{g}) \rightarrow 2\text{H}_2\text{O}(\text{l})$
(B) $2\text{C}(\text{g}) + 3\text{H}_2(\text{g}) \rightarrow \text{C}_2\text{H}_6(\text{g})$
(C) $\frac{3}{2}\text{O}_2(\text{g}) \rightarrow \text{O}_3(\text{g})$
(D) $\frac{1}{8}\text{S}_8(\text{s}) + \text{O}_2(\text{g}) \rightarrow \text{SO}_2(\text{g})$

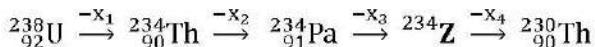
Q.8 Which of the following statement(s) is(are) correct regarding the root mean square speed (u_{rms}) and average translational kinetic energy (ε_{av}) of a molecule in a gas at equilibrium ?

(A) u_{rms} is doubled when its temperature is increased four times
(B) ε_{av} is doubled when its temperature is increased four times
(C) ε_{av} at a given temperature does not depend on its molecular mass
(D) u_{rms} is inversely proportional to the square root of its molecular mass

Q.9 Each of the following options contains a set of four molecules. Identify the option(s) where all four molecules possess permanent dipole moment at room temperature.

(A) BeCl_2 , CO_2 , BCl_3 , CHCl_3
(B) NO_2 , NH_3 , POCl_3 , CH_3Cl
(C) BF_3 , O_3 , SF_6 , XeF_6
(D) SO_2 , $\text{C}_6\text{H}_5\text{Cl}$, H_2Se , BrF_5

Q.10 In the decay sequence,



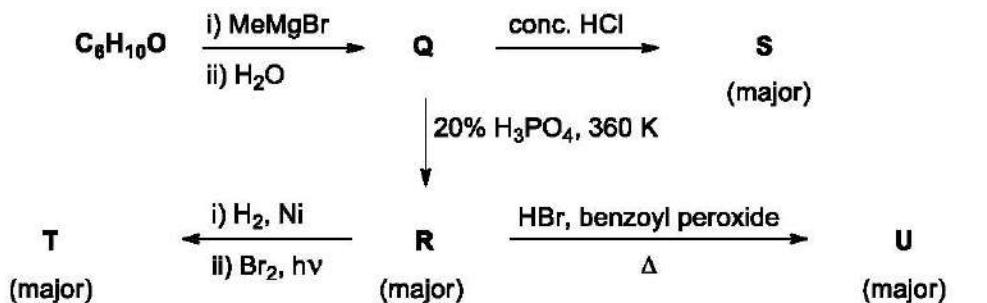
x_1, x_2, x_3 and x_4 are particles/radiation emitted by the respective isotopes. The correct option(s) is(are)

- (A) x_1 will deflect towards negatively charged plate
- (B) x_2 is β^-
- (C) x_3 is γ -ray
- (D) Z is an isotope of uranium

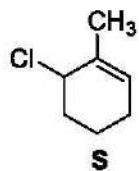
Q.11 Which of the following statement(s) is(are) true?

- (A) Monosaccharides cannot be hydrolysed to give polyhydroxy aldehydes and ketones
- (B) Oxidation of glucose with bromine water gives glutamic acid
- (C) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose
- (D) The two six-membered cyclic hemiacetal forms of D-(+)-glucose are called anomers

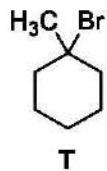
Q.12 Choose the correct option(s) for the following set of reactions



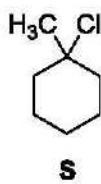
(A)



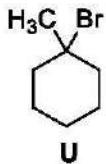
(B)



(c)



(D)



$$\begin{array}{c} \text{CH}_3 \\ | \\ \text{C}_6\text{H}_11-\text{CH}-\text{Cl} \\ | \\ \text{S} \end{array}$$

7

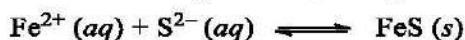
SECTION 3 (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +3 If ONLY the correct numerical value is entered;
 Zero Marks : 0 In all other cases.

Q.13 Among B_2H_6 , $\text{B}_3\text{N}_3\text{H}_6$, N_2O , N_2O_4 , $\text{H}_2\text{S}_2\text{O}_3$ and $\text{H}_2\text{S}_2\text{O}_8$, the total number of molecules containing covalent bond between two atoms of the same kind is _____

Q.14 At 143 K, the reaction of XeF_4 with O_2F_2 produces a xenon compound Y. The total number of lone pair(s) of electrons present on the whole molecule of Y is _____

Q.15 For the following reaction, the equilibrium constant K_c at 298 K is 1.6×10^{17} .



When equal volumes of 0.06 M $\text{Fe}^{2+}(\text{aq})$ and 0.2 M $\text{S}^{2-}(\text{aq})$ solutions are mixed, the equilibrium concentration of $\text{Fe}^{2+}(\text{aq})$ is found to be $Y \times 10^{-17}$ M. The value of Y is _____

Q.16 On dissolving 0.5 g of a non-volatile non-ionic solute to 39 g of benzene, its vapor pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of benzene (in K) upon addition of the solute is _____
 (Given data: Molar mass and the molal freezing point depression constant of benzene are 78 g mol^{-1} and 5.12 K kg mol^{-1} , respectively)

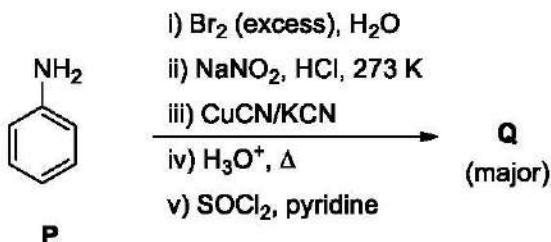
Q.17 Consider the kinetic data given in the following table for the reaction $\text{A} + \text{B} + \text{C} \rightarrow \text{Product}$.

Experiment No.	[A] (mol dm^{-3})	[B] (mol dm^{-3})	[C] (mol dm^{-3})	Rate of reaction (mol $\text{dm}^{-3} \text{s}^{-1}$)
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

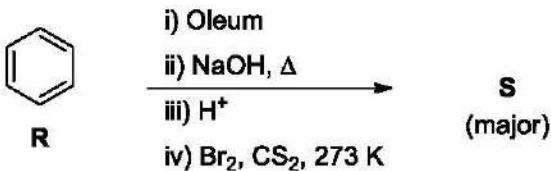
The rate of the reaction for $[\text{A}] = 0.15 \text{ mol dm}^{-3}$, $[\text{B}] = 0.25 \text{ mol dm}^{-3}$ and $[\text{C}] = 0.15 \text{ mol dm}^{-3}$ is found to be $Y \times 10^{-5} \text{ mol dm}^{-3} \text{s}^{-1}$. The value of Y is _____

Q.18 Schemes 1 and 2 describe the conversion of P to Q and R to S, respectively. Scheme 3 describes the synthesis of T from Q and S. The total number of Br atoms in a molecule of T is _____

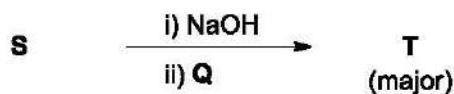
Scheme 1:



Scheme 2:



Scheme 3:



**JEE (ADVANCED) 2019 PAPER 1
PART-III MATHEMATICS**

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.1 Let S be the set of all complex numbers z satisfying $|z - 2 + i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{ \frac{1}{|z - 1|} : z \in S \right\}$, then the principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is

(A) $-\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

Q.2 Let

$$M = \begin{bmatrix} \sin^4 \theta & -1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} = \alpha I + \beta M^{-1},$$

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If

α^* is the minimum of the set $\{\alpha(\theta): \theta \in [0, 2\pi)\}$ and

β^* is the minimum of the set $\{\beta(\theta): \theta \in [0, 2\pi]\}$,

then the value of $\alpha^* + \beta^*$ is

(A) $-\frac{37}{16}$ (B) $-\frac{31}{16}$ (C) $-\frac{29}{16}$ (D) $-\frac{17}{16}$

Q.3 A line $y = mx + 1$ intersects the circle $(x - 3)^2 + (y + 2)^2 = 25$ at the points P and Q . If the midpoint of the line segment PQ has x -coordinate $-\frac{3}{5}$, then which one of the following options is correct?

(A) $-3 \leq m < -1$ (B) $2 \leq m < 4$
 (C) $4 \leq m < 6$ (D) $6 \leq m < 8$

Q.4 The area of the region $\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$ is

(A) $16 \log_e 2 - \frac{14}{3}$ (B) $8 \log_e 2 - \frac{14}{3}$
(C) $16 \log_e 2 - 6$ (D) $8 \log_e 2 - \frac{7}{3}$

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -1	In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
 choosing ONLY (A) and (B) will get +2 marks;
 choosing ONLY (A) and (D) will get +2 marks;
 choosing ONLY (B) and (D) will get +2 marks;
 choosing ONLY (A) will get +1 mark;
 choosing ONLY (B) will get +1 mark;
 choosing ONLY (D) will get +1 mark;
 choosing no option (i.e. the question is unanswered) will get 0 marks; and
 choosing any other combination of options will get -1 mark.

Q.5 Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n , define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1,$$

$$b_1 = 1 \quad \text{and} \quad b_n = a_{n-1} + a_{n+1}, \quad n \geq 2.$$

Then which of the following options is/are correct?

(A) $a_1 + a_2 + a_3 + \dots + a_n = a_{n+2} - 1$ for all $n \geq 1$

(B) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(C) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

(D) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

Q.6 Let

$$M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix} \quad \text{and} \quad \text{adj } M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

where a and b are real numbers. Which of the following options is/are correct?

- (A) $a + b = 3$
- (B) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$
- (C) $\det(\text{adj } M^2) = 81$
- (D) If $M \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

Q.7 There are three bags B_1 , B_2 and B_3 . The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1 , B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

- (A) Probability that the chosen ball is green, given that the selected bag is B_3 , equals $\frac{3}{8}$
- (B) Probability that the chosen ball is green equals $\frac{39}{80}$
- (C) Probability that the selected bag is B_3 , given that the chosen ball is green, equals $\frac{5}{13}$
- (D) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

Q.8 In a non-right-angled triangle ΔPQR , let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S , the perpendicular from P meets the side QR at E , and RS and PE intersect at O . If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

(A) Length of $RS = \frac{\sqrt{7}}{2}$
 (B) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$
 (C) Length of $OE = \frac{1}{6}$
 (D) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2} (2 - \sqrt{3})$

Q.9 Define the collections $\{E_1, E_2, E_3, \dots\}$ of ellipses and $\{R_1, R_2, R_3, \dots\}$ of rectangles as follows:

$$E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1;$$

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1 ;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1} , $n > 1$;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n , $n > 1$.

Then which of the following options is/are correct?

(A) The eccentricities of E_{18} and E_{19} are NOT equal
 (B) $\sum_{n=1}^N (\text{area of } R_n) < 24$, for each positive integer N
 (C) The length of latus rectum of E_9 is $\frac{1}{6}$
 (D) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

Q.10 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \leq x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \leq x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \geq 3. \end{cases}$$

Then which of the following options is/are correct?

- (A) f is increasing on $(-\infty, 0)$
- (B) f' has a local maximum at $x = 1$
- (C) f is onto
- (D) f' is NOT differentiable at $x = 1$

Q.11 Let Γ denote a curve $y = y(x)$ which is in the first quadrant and let the point $(1,0)$ lie on it. Let the tangent to Γ at a point P intersect the y -axis at Y_P . If PY_P has length 1 for each point P on Γ , then which of the following options is/are correct?

- (A) $y = \log_e\left(\frac{1 + \sqrt{1 - x^2}}{x}\right) - \sqrt{1 - x^2}$
- (B) $xy' + \sqrt{1 - x^2} = 0$
- (C) $y = -\log_e\left(\frac{1 + \sqrt{1 - x^2}}{x}\right) + \sqrt{1 - x^2}$
- (D) $xy' - \sqrt{1 - x^2} = 0$

Q.12 Let L_1 and L_2 denote the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in \mathbb{R} \text{ and}$$
$$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \mathbb{R}$$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ?

- (A) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$
- (B) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$
- (C) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$
- (D) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

SECTION 3 (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +3 If ONLY the correct numerical value is entered;
 Zero Marks : 0 In all other cases.

Q.13 Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set

$$\{|a + b\omega + c\omega^2|^2 : a, b, c \text{ distinct non-zero integers}\}$$

equals _____

Q.14 Let $AP(a; d)$ denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If

$$AP(1; 3) \cap AP(2; 5) \cap AP(3; 7) = AP(a; d)$$

then $a + d$ equals _____

Q.15 Let S be the sample space of all 3×3 matrices with entries from the set $\{0, 1\}$. Let the events E_1 and E_2 be given by

$$E_1 = \{A \in S : \det A = 0\} \text{ and}$$

$$E_2 = \{A \in S : \text{sum of entries of } A \text{ is 7}\}.$$

If a matrix is chosen at random from S , then the conditional probability $P(E_1|E_2)$ equals _____

Q.16 Let the point B be the reflection of the point $A(2, 3)$ with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T . If C is the point of intersection of T and the line passing through A and B , then the length of the line segment AC is _____

Q.17 If

$$I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)}$$

then $27 I^2$ equals _____

Q.18 Three lines are given by

$$\vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R}$$

$$\vec{r} = \mu (\hat{i} + \hat{j}), \mu \in \mathbb{R} \text{ and}$$

$$\vec{r} = \nu (\hat{i} + \hat{j} + \hat{k}), \nu \in \mathbb{R}.$$

Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals _____

JEE (ADVANCED) 2019 PAPER 2

PART-I PHYSICS

SECTION 1 (Maximum Marks: 32)

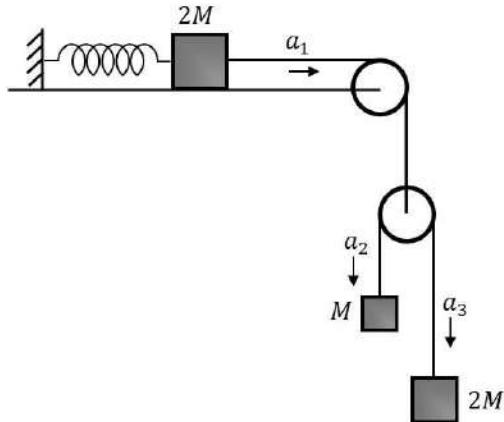
- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	: +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
 choosing ONLY (A) and (B) will get +2 marks;
 choosing ONLY (A) and (D) will get +2 marks;
 choosing ONLY (B) and (D) will get +2 marks;
 choosing ONLY (A) will get +1 mark;
 choosing ONLY (B) will get +1 mark;
 choosing ONLY (D) will get +1 mark;
 choosing no option (i.e. the question is unanswered) will get 0 marks; and
 choosing any other combination of options will get -1 mark.

Q.1 A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical?
 [g is the acceleration due to gravity]

- (A) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$
- (B) The angular acceleration of the rod will be $\frac{2g}{L}$
- (C) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$
- (D) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

Q.2 A block of mass $2M$ is attached to a massless spring with spring-constant k . This block is connected to two other blocks of masses M and $2M$ using two massless pulleys and strings. The accelerations of the blocks are a_1 , a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0 . Which of the following option(s) is/are correct?
 [g is the acceleration due to gravity. Neglect friction]



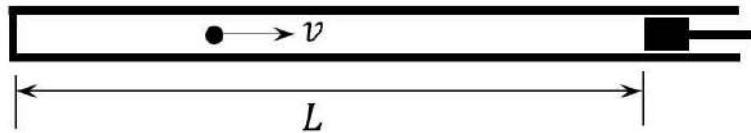
(A) $x_0 = \frac{4Mg}{k}$

(B) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(C) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

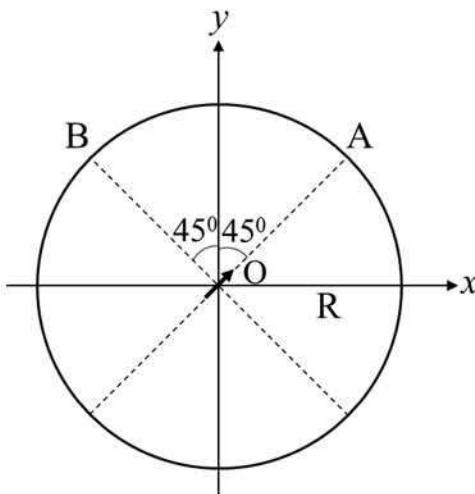
(D) $a_2 - a_1 = a_1 - a_3$

Q.3 A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V \ll \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?



- (A) The rate at which the particle strikes the piston is v/L
- (B) After each collision with the piston, the particle speed increases by $2V$
- (C) If the piston moves inward by dL , the particle speed increases by $2v \frac{dL}{L}$
- (D) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

Q.4 An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i} + \hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0 . If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are: (ϵ_0 is permittivity of free space. $R \gg$ dipole size)



(A) $R = \left(\frac{p_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$

(B) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i} + \hat{j})$

(C) Total electric field at point B is $\vec{E}_B = 0$

(D) The magnitude of total electric field on any two points of the circle will be same.

Q.5 A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0 , volume V_0 , and temperature T_0 . If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, (Given $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

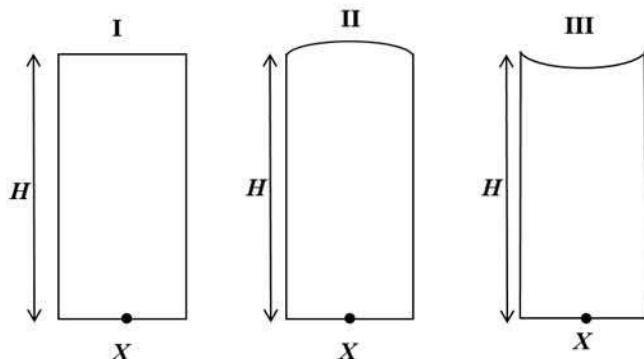
(A) The work $|W|$ done during the process is $13RT_0$

(B) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(C) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

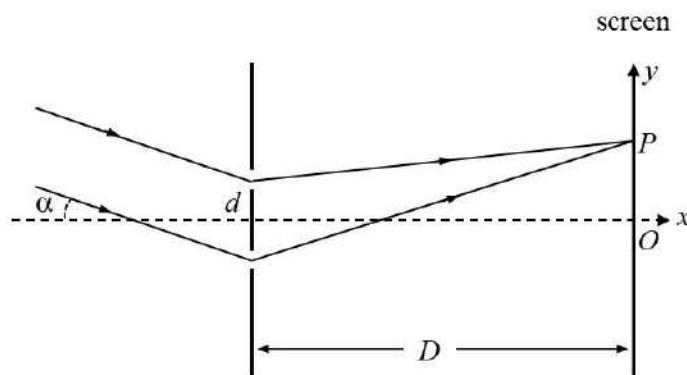
(D) Adiabatic constant of the gas mixture is 1.6

Q.6 Three glass cylinders of equal height $H = 30$ cm and same refractive index $n = 1.5$ are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same ($R = 3$ m). If H_1 , H_2 , and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are:



(A) $H_2 > H_1$
 (B) $H_3 > H_1$
 (C) $H_2 > H_3$
 (D) $0.8 \text{ cm} < (H_2 - H_1) < 0.9 \text{ cm}$

Q.7 In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct?



(A) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O .

(B) For $\alpha = 0$, there will be constructive interference at point P .

(C) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P .

(D) Fringe spacing depends on α .

Q.8 A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state $n = 1$ to the state $n = 4$. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e . Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e , respectively. If $\lambda_a/\lambda_e = \frac{1}{5}$, which of the option(s) is/are correct?
 [Use $hc = 1242 \text{ eV nm}$; $1 \text{ nm} = 10^{-9} \text{ m}$, \hbar and c are Planck's constant and speed of light, respectively]

(A) $m = 2$

(B) $\lambda_e = 418 \text{ nm}$

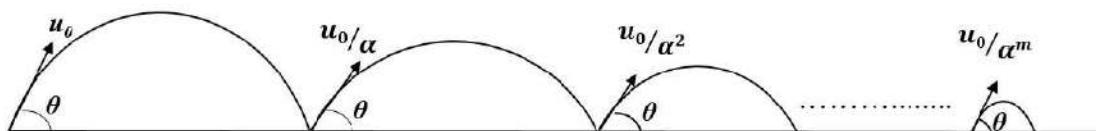
(C) $\Delta p_a/\Delta p_e = \frac{1}{2}$

(D) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is $\frac{1}{4}$

SECTION 2 (Maximum Marks: 18)

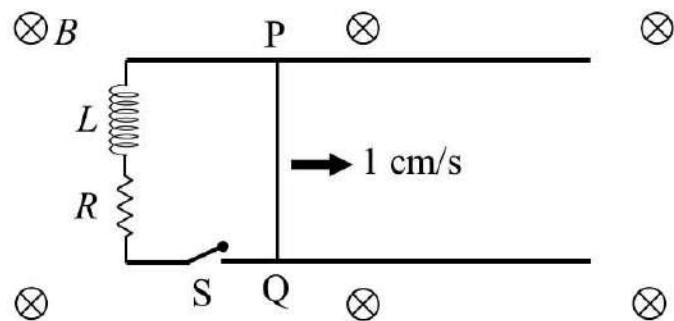
- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +3 If ONLY the correct numerical value is entered;
 Zero Marks : 0 In all other cases.

Q.9 A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0 . For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1 . After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α . Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8 V_1$, the value of α is _____.

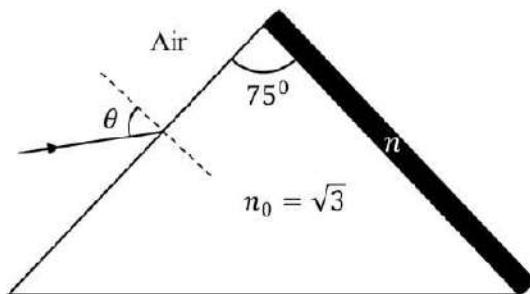


Q.10 A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor $L = 1$ mH and a resistance $R = 1 \Omega$ as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field $B = 1$ T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is _____.

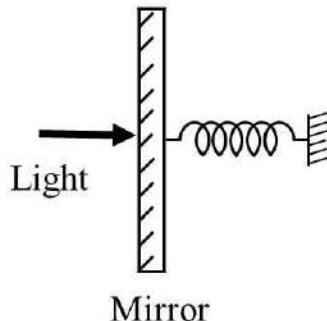
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given: $e^{-1} = 0.37$, where e is base of the natural logarithm]



Q.11 A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \leq 60^\circ$. The value of n^2 is _____.



Q.12 A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24} \text{ m}^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6} \text{ m}$ strike the mirror simultaneously at normal incidence such that the mirror gets displaced by $1 \mu\text{m}$. If the value of N is $x \times 10^{12}$, then the value of x is _____.
 [Consider the spring as massless]



Q.13 Suppose a $^{226}_{88}\text{Ra}$ nucleus at rest and in ground state undergoes α -decay to a $^{222}_{86}\text{Rn}$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV . $^{222}_{86}\text{Rn}$ nucleus then goes to its ground state by γ -decay. The energy of the emitted γ photon is ____ keV.
 [Given: atomic mass of $^{226}_{88}\text{Ra} = 226.005 \text{ u}$, atomic mass of $^{222}_{86}\text{Rn} = 222.000 \text{ u}$, atomic mass of α particle = 4.000 u , $1 \text{ u} = 931 \text{ MeV}/c^2$, c is speed of the light]

Q.14 An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is _____.

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If ONLY the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
Negative Marks : -1 In all other cases.

Answer Q.15 and Q.16 by appropriately matching the lists based on the information given in the paragraph

A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length μ , 2μ , 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0 .

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	List-II
(I) String-1 (μ)	(P) 1
(II) String-2 (2μ)	(Q) $1/2$
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) $3/16$
	(U) $1/16$

Q.15 If the tension in each string is T_0 , the correct match for the highest fundamental frequency in f_0 units will be,

(A) I \rightarrow P, II \rightarrow Q, III \rightarrow T, IV \rightarrow S (B) I \rightarrow P, II \rightarrow R, III \rightarrow S, IV \rightarrow Q
 (C) I \rightarrow Q, II \rightarrow S, III \rightarrow R, IV \rightarrow P (D) I \rightarrow Q, II \rightarrow P, III \rightarrow R, IV \rightarrow T

Q.16 The length of the strings 1, 2, 3 and 4 are kept fixed at L_0 , $\frac{3L_0}{2}$, $\frac{5L_0}{4}$, and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3, and 4 are vibrated at their 1st, 3rd, 5th, and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be,

(A) I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow U (B) I \rightarrow P, II \rightarrow Q, III \rightarrow T, IV \rightarrow U
 (C) I \rightarrow P, II \rightarrow Q, III \rightarrow R, IV \rightarrow T (D) I \rightarrow T, II \rightarrow Q, III \rightarrow R, IV \rightarrow U

Answer Q.17 and Q.18 by appropriately matching the lists based on the information given in the paragraph

In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants .

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

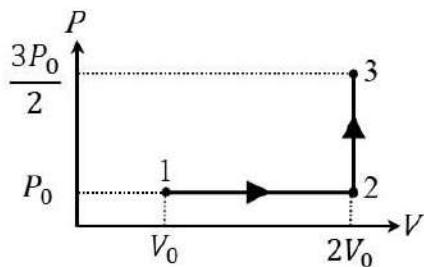
List-I

- (I) Work done by the system in process 1→2→3
- (II) Change in internal energy in process 1→2→3
- (III) Heat absorbed by the system in process 1→2→3
- (IV) Heat absorbed by the system in process 1→2

List-II

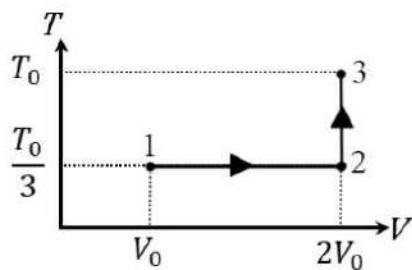
- (P) $\frac{1}{3}RT_0 \ln 2$
- (Q) $\frac{1}{3}RT_0$
- (R) RT_0
- (S) $\frac{4}{3}RT_0$
- (T) $\frac{1}{3}RT_0(3 + \ln 2)$
- (U) $\frac{5}{6}RT_0$

Q.17 If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV -diagram with $P_0 V_0 = \frac{1}{3} R T_0$, the correct match is,



(A) I \rightarrow Q, II \rightarrow R, III \rightarrow P, IV \rightarrow U
 (B) I \rightarrow Q, II \rightarrow R, III \rightarrow S, IV \rightarrow U
 (C) I \rightarrow S, II \rightarrow R, III \rightarrow Q, IV \rightarrow T
 (D) I \rightarrow Q, II \rightarrow S, III \rightarrow R, IV \rightarrow U

Q.18 If the process on one mole of monatomic ideal gas is as shown in the TV -diagram with $P_0 V_0 = \frac{1}{3} R T_0$, the correct match is,



(A) I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow S
 (B) I \rightarrow P, II \rightarrow T, III \rightarrow Q, IV \rightarrow T
 (C) I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow P
 (D) I \rightarrow S, II \rightarrow T, III \rightarrow Q, IV \rightarrow U

JEE (ADVANCED) 2019 PAPER 2

PART-II CHEMISTRY

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -1	In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
 choosing ONLY (A) and (B) will get +2 marks;
 choosing ONLY (A) and (D) will get +2 marks;
 choosing ONLY (B) and (D) will get +2 marks;
 choosing ONLY (A) will get +1 mark;
 choosing ONLY (B) will get +1 mark;
 choosing ONLY (D) will get +1 mark;
 choosing no option (i.e. the question is unanswered) will get 0 marks; and
 choosing any other combination of options will get -1 mark.

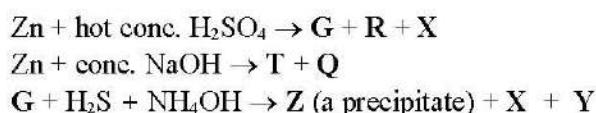
Q.1 The cyanide process of gold extraction involves leaching out gold from its ore with CN^- in the presence of Q in water to form R. Subsequently, R is treated with T to obtain Au and Z. Choose the correct option(s)

- (A) Q is O_2
- (B) T is Zn
- (C) Z is $[\text{Zn}(\text{CN})_4]^{2-}$
- (D) R is $[\text{Au}(\text{CN})_4]^-$

Q.2 With reference to *aqua regia*, choose the correct option(s)

- (A) *Aqua regia* is prepared by mixing conc. HCl and conc. HNO_3 in 3:1 (v/v) ratio
- (B) Reaction of gold with *aqua regia* produces an anion having Au in +3 oxidation state
- (C) Reaction of gold with *aqua regia* produces NO_2 in the absence of air
- (D) The yellow colour of *aqua regia* is due to the presence of NOCl and Cl_2

Q.3 Consider the following reactions (unbalanced)



Choose the correct option(s)

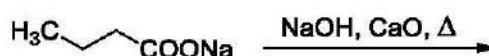
- (A) Z is dirty white in colour
- (B) The oxidation state of Zn in T is +1
- (C) R is a V-shaped molecule
- (D) Bond order of Q is 1 in its ground state

Q.4 The ground state energy of hydrogen atom is -13.6 eV . Consider an electronic state Ψ of He^+ whose energy, azimuthal quantum number and magnetic quantum number are -3.4 eV , 2 and 0, respectively. Which of the following statement(s) is(are) true for the state Ψ ?

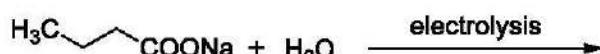
- (A) It is a $4d$ state
- (B) It has 2 angular nodes
- (C) It has 3 radial nodes
- (D) The nuclear charge experienced by the electron in this state is less than $2e$, where e is the magnitude of the electronic charge

Q.5 Which of the following reactions produce(s) propane as a major product?

(A)



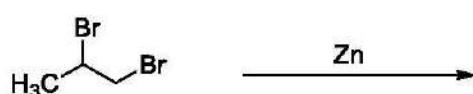
(B)



(C)

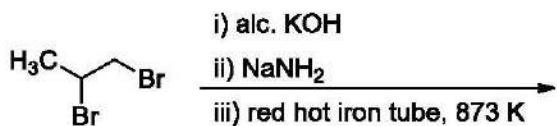


(D)



Q.6 Choose the correct option(s) that give(s) an aromatic compound as the major product

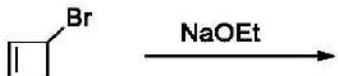
(A)



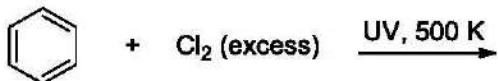
(B)



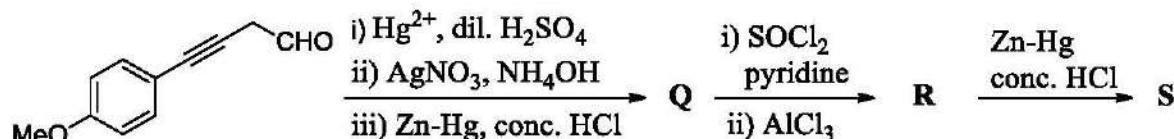
(C)



(D)

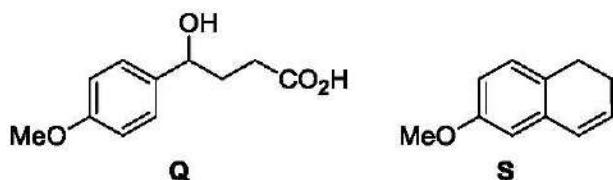


Q.7 Choose the correct option(s) for the following reaction sequence



Consider Q, R and S as major products

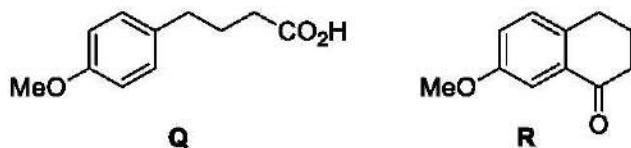
(A)



(B)



(C)



(D)



Q.8 Choose the correct option(s) from the following

(A) Natural rubber is polyisoprene containing *trans* alkene units

(B) Nylon-6 has amide linkages

(C) Teflon is prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high pressure

(D) Cellulose has only α -D-glucose units that are joined by glycosidic linkages

SECTION 2 (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If ONLY the correct numerical value is entered;
Zero Marks : 0 In all other cases.

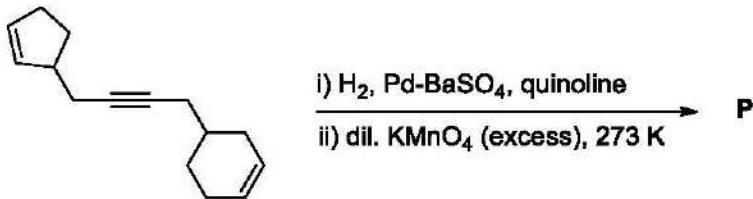
Q.9 The amount of water produced (in g) in the oxidation of 1 mole of rhombic sulphur by conc. HNO_3 to a compound with the highest oxidation state of sulphur is _____
(Given data: Molar mass of water = 18 g mol^{-1})

Q.10 Total number of *cis* N–Mn–Cl bond angles (that is, Mn–N and Mn–Cl bonds in *cis* positions) present in a molecule of *cis*- $[\text{Mn}(\text{en})_2\text{Cl}_2]$ complex is _____ ($\text{en} = \text{NH}_2\text{CH}_2\text{CH}_2\text{NH}_2$)

Q.11 The decomposition reaction $2\text{N}_2\text{O}_5(g) \xrightarrow{\Delta} 2\text{N}_2\text{O}_4(g) + \text{O}_2(g)$ is started in a closed cylinder under isothermal isochoric condition at an initial pressure of 1 atm. After $\text{Y} \times 10^3$ s, the pressure inside the cylinder is found to be 1.45 atm. If the rate constant of the reaction is $5 \times 10^{-4} \text{ s}^{-1}$, assuming ideal gas behavior, the value of Y is _____

Q.12 The mole fraction of urea in an aqueous urea solution containing 900 g of water is 0.05. If the density of the solution is 1.2 g cm^{-3} , the molarity of urea solution is _____
(Given data: Molar masses of urea and water are 60 g mol^{-1} and 18 g mol^{-1} , respectively)

Q.13 Total number of hydroxyl groups present in a molecule of the major product P is _____



Q.14 Total number of isomers, considering both structural and stereoisomers, of cyclic ethers with the molecular formula $\text{C}_4\text{H}_8\text{O}$ is _____

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
Negative Marks : -1 In all other cases.

Answer Q.15 and Q.16 by appropriately matching the lists based on the information given in the paragraph

Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following, List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n .

List-I

List-II

(I) Radius of the n^{th} orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the n^{th} orbit	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the n^{th} orbit	(R) $\propto n^0$
(IV) Potential energy of the electron in the n^{th} orbit	(S) $\propto n^1$
	(T) $\propto n^2$
	(U) $\propto n^{1/2}$

Q.15 Which of the following options has the correct combination considering List-I and List-II?

(A) (I), (P) (B) (I), (T) (C) (II), (Q) (D) (II), (R)

Q.16 Which of the following options has the correct combination considering List-I and List-II?

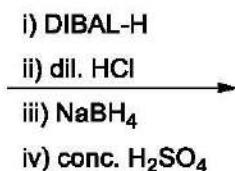
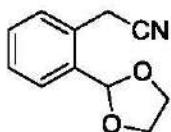
(A) (III), (P) (B) (III), (S) (C) (IV), (Q) (D) (IV), (U)

Answer Q.17 and Q.18 by appropriately matching the lists based on the information given in the paragraph

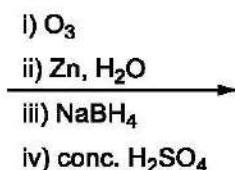
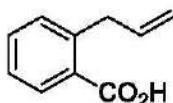
List-I includes starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and/or final products from the reactions of List-I.

List-I

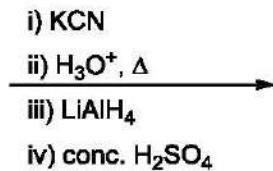
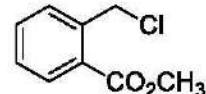
(I)



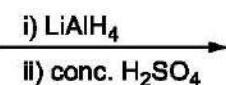
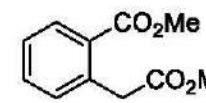
(II)



(III)

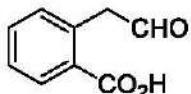


(IV)

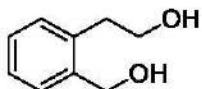


List-II

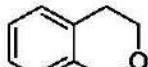
(P)



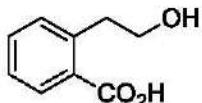
(Q)



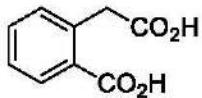
(R)



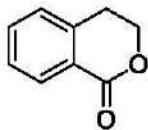
(S)



(T)



(U)



Q.17 Which of the following options has correct combination considering List-I and List-II?

(A) (I), (Q), (T), (U)	(B) (II), (P), (S), (T)
(C) (II), (P), (S), (U)	(D) (I), (S), (Q), (R)

Q.18 Which of the following options has correct combination considering List-I and List-II?

(A) (III), (S), (R)	(B) (IV), (Q), (U)
(C) (III), (T), (U)	(D) (IV), (Q), (R)

JEE (ADVANCED) 2019 PAPER 2

PART-III MATHEMATICS

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
 - Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
 - Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
 - Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
 - Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
 - Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
 - Negative Marks : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get -1 mark.

Q.1 Let

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{and } X = \sum_{k=1}^6 P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$$

where P_k^T denotes the transpose of the matrix P_k . Then which of the following options is/are correct?

- (A) If $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$
- (B) X is a symmetric matrix
- (C) The sum of diagonal entries of X is 18
- (D) $X - 30I$ is an invertible matrix

Q.2 Let $x \in \mathbb{R}$ and let

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \quad Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = P Q P^{-1}.$$

Then which of the following options is/are correct?

(A) There exists a real number x such that $PQ = QP$

(B) $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in \mathbb{R}$

(C) For $x = 0$, if $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then $a + b = 5$

(D) For $x = 1$, there exists a unit vector $\alpha \hat{i} + \beta \hat{j} + \gamma \hat{k}$ for which $R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Q.3 For non-negative integers n , let

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$, which of the following options is/are correct?

(A) $f(4) = \frac{\sqrt{3}}{2}$

(B) $\lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$

(C) If $\alpha = \tan(\cos^{-1}f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

(D) $\sin(7 \cos^{-1}f(5)) = 0$

Q.4 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. We say that f has

PROPERTY 1 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct?

(A) $f(x) = x $ has PROPERTY 1	(B) $f(x) = x^{2/3}$ has PROPERTY 1
(C) $f(x) = x x $ has PROPERTY 2	(D) $f(x) = \sin x$ has PROPERTY 2

Q.5 Let

$$f(x) = \frac{\sin \pi x}{x^2}, \quad x > 0.$$

Let $x_1 < x_2 < x_3 < \dots < x_n < \dots$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < \dots < y_n < \dots$ be all the points of local minimum of f .

Then which of the following options is/are correct?

(A) $x_1 < y_1$	(B) $x_{n+1} - x_n > 2$ for every n
(C) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n	(D) $ x_n - y_n > 1$ for every n

Q.6 For $a \in \mathbb{R}$, $|a| > 1$, let

$$\lim_{n \rightarrow \infty} \left(\frac{1 + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{n^{7/3} \left(\frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + \dots + \frac{1}{(an+n)^2} \right)} \right) = 54.$$

Then the possible value(s) of a is/are

(A) -9	(B) -6	(C) 7	(D) 8
--------	--------	-------	-------

Q.7 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = (x-1)(x-2)(x-5)$. Define

$$F(x) = \int_0^x f(t) dt, \quad x > 0.$$

Then which of the following options is/are correct?

- (A) F has a local minimum at $x = 1$
- (B) F has a local maximum at $x = 2$
- (C) F has two local maxima and one local minimum in $(0, \infty)$
- (D) $F(x) \neq 0$ for all $x \in (0, 5)$

Q.8 Three lines

$$L_1: \quad \vec{r} = \lambda \hat{i}, \quad \lambda \in \mathbb{R},$$

$$L_2: \quad \vec{r} = \hat{k} + \mu \hat{j}, \quad \mu \in \mathbb{R} \text{ and}$$

$$L_3: \quad \vec{r} = \hat{i} + \hat{j} + \nu \hat{k}, \quad \nu \in \mathbb{R}$$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear?

- (A) $\hat{k} - \frac{1}{2} \hat{j}$
- (B) \hat{k}
- (C) $\hat{k} + \frac{1}{2} \hat{j}$
- (D) $\hat{k} + \hat{j}$

SECTION 2 (Maximum Marks: 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +3 If ONLY the correct numerical value is entered;
 Zero Marks : 0 In all other cases.

Q.9 Suppose

$$\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^n C_k k^2 \\ \sum_{k=0}^n {}^n C_k k & \sum_{k=0}^n {}^n C_k 3^k \end{bmatrix} = 0$$

holds for some positive integer n . Then $\sum_{k=0}^n \frac{{}^n C_k}{k+1}$ equals __

Q.10 Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is __

Q.11 Let $|X|$ denote the number of elements in a set X . Let $S = \{1, 2, 3, 4, 5, 6\}$ be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S , then the number of ordered pairs (A, B) such that $1 \leq |B| < |A|$, equals __

Q.12 The value of

$$\sec^{-1} \left(\frac{1}{4} \sum_{k=0}^{10} \sec \left(\frac{7\pi}{12} + \frac{k\pi}{2} \right) \sec \left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2} \right) \right)$$

 in the interval $\left[-\frac{\pi}{4}, \frac{3\pi}{4} \right]$ equals ___

Q.13 The value of the integral

$$\int_0^{\pi/2} \frac{3 \sqrt{\cos \theta}}{(\sqrt{\cos \theta} + \sqrt{\sin \theta})^5} d\theta$$

equals ___

 Q.14 Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, $\alpha, \beta \in \mathbb{R}$. If the projection of \vec{c} on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ equals ___

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
Negative Marks : -1 In all other cases.

Answer Q.15 and Q.16 by appropriately matching the lists based on the information given in the paragraph.

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order:

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\},$$

$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List – I contains the sets X , Y , Z and W . List – II contains some information regarding these sets.

List-I

(I) X

List-II

(P) $\supseteq \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$

(II) Y

(Q) an arithmetic progression

(III) Z

(R) NOT an arithmetic progression

(IV) W

(S) $\supseteq \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$

(T) $\supseteq \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$

(U) $\supseteq \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$

Q.15 Which of the following is the only CORRECT combination?

(A) (I), (P), (R) (B) (II), (Q), (T) (C) (I), (Q), (U) (D) (II), (R), (S)

Q.16 Which of the following is the only CORRECT combination?

(A) (III), (R), (U) (C) (III), (P), (Q), (U)	(B) (IV), (P), (R), (S) (D) (IV), (Q), (T)
---	---

Answer Q.17 and Q.18 by appropriately matching the lists based on the information given in the paragraph.

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y . Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions:

- (i) centre of C_3 is collinear with the centres of C_1 and C_2 ,
- (ii) C_1 and C_2 both lie inside C_3 , and
- (iii) C_3 touches C_1 at M and C_2 at N .

Let the line through X and Y intersect C_3 at Z and W , and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8ay$.

There are some expressions given in the List-I whose values are given in List-II below:

List-I

- (I) $2h + k$
- (II) $\frac{\text{Length of } ZW}{\text{Length of } XY}$
- (III) $\frac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$
- (IV) α

List-II

- (P) 6
- (Q) $\sqrt{6}$
- (R) $\frac{5}{4}$
- (S) $\frac{21}{5}$
- (T) $2\sqrt{6}$
- (U) $\frac{10}{3}$

Q.17 Which of the following is the only CORRECT combination?

(A) (I), (S) (B) (I), (U) (C) (II), (Q) (D) (II), (T)

Q.18 Which of the following is the only INCORRECT combination?

(A) (I), (P) (B) (IV), (U) (C) (III), (R) (D) (IV), (S)