

## PART I : PHYSICS

### SECTION 1 (Maximum Marks: 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

<i>Full Marks</i>	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
<i>Partial Marks</i>	: +1	For darkening a bubble corresponding <b>to each correct option</b> , provided NO incorrect option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks</i>	: -2	In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.1 A flat plate is moving normal to its plane through a gas under the action of a constant force  $F$ . The gas is kept at a very low pressure. The speed of the plate  $v$  is much less than the average speed  $u$  of the gas molecules. Which of the following options is/are true?

- [A] The pressure difference between the leading and trailing faces of the plate is proportional to  $uv$
- [B] The resistive force experienced by the plate is proportional to  $v$
- [C] The plate will continue to move with constant non-zero acceleration, at all times
- [D] At a later time the external force  $F$  balances the resistive force

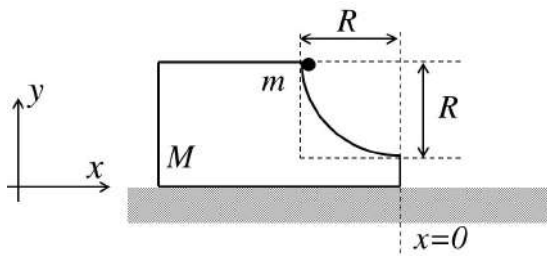
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**Answer for the above question**

**Ans for Q.1: (A), (B), and (D)**

- Q.2 A block of mass  $M$  has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at  $x = 0$ , in a *co-ordinate system fixed to the table*. A point mass  $m$  is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is  $x$  and the velocity is  $v$ . At that instant, which of the following options is/are correct?



- [A] The position of the point mass  $m$  is:  $x = -\sqrt{2} \frac{mR}{M+m}$
- [B] The velocity of the point mass  $m$  is:  $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$
- [C] The  $x$  component of displacement of the center of mass of the block  $M$  is:  $-\frac{mR}{M+m}$
- [D] The velocity of the block  $M$  is:  $V = -\frac{m}{M} \sqrt{2gR}$

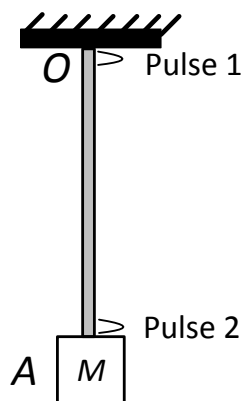
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**Answer for the above question**

**Ans for Q.2: (B) and (C)**

- Q.3 A block  $M$  hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at  $O$ . A transverse wave pulse (Pulse 1) of wavelength  $\lambda_0$  is produced at point  $O$  on the rope. The pulse takes time  $T_{OA}$  to reach point  $A$ . If the wave pulse of wavelength  $\lambda_0$  is produced at point  $A$  (Pulse 2) without disturbing the position of  $M$  it takes time  $T_{AO}$  to reach point  $O$ . Which of the following options is/are correct?



- [A] The time  $T_{AO} = T_{OA}$
- [B] The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
- [C] The wavelength of Pulse 1 becomes longer when it reaches point  $A$
- [D] The velocity of any pulse along the rope is independent of its frequency and wavelength

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Space for rough work

**Answer for the above question**

**Ans for Q.3: (A) and (D)**

Q.4 A human body has a surface area of approximately  $1 \text{ m}^2$ . The normal body temperature is  $10 \text{ K}$  above the surrounding room temperature  $T_0$ . Take the room temperature to be  $T_0 = 300 \text{ K}$ . For  $T_0 = 300 \text{ K}$ , the value of  $\sigma T_0^4 = 460 \text{ Wm}^{-2}$  (where  $\sigma$  is the Stefan-Boltzmann constant). Which of the following options is/are correct?

- [A] The amount of energy radiated by the body in 1 second is close to 60 Joules
- [B] If the surrounding temperature reduces by a small amount  $\Delta T_0 \ll T_0$ , then to maintain the same body temperature the same (living) human being needs to radiate  $\Delta W = 4\sigma T_0^3 \Delta T_0$  more energy per unit time
- [C] Reducing the exposed surface area of the body (*e.g.* by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
- [D] If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

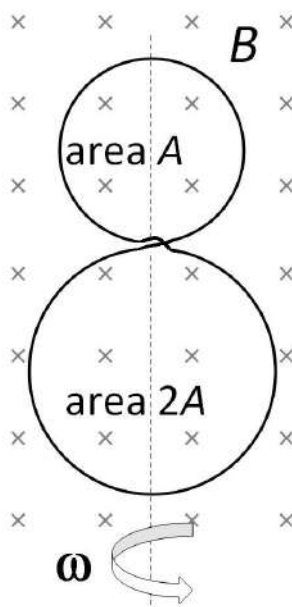
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**Answer for the above question**

<b>Ans for Q.4: (C)</b>
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- Q.5 A circular insulated copper wire loop is twisted to form two loops of area  $A$  and  $2A$  as shown in the figure. *At the point of crossing the wires remain electrically insulated from each other.* The entire loop lies in the plane (of the paper). A uniform magnetic field  $\vec{B}$  points into the plane of the paper. At  $t = 0$ , the loop starts rotating about the common diameter as axis with a constant angular velocity  $\omega$  in the magnetic field. Which of the following options is/are correct?



- [A] The  $emf$  induced in the loop is proportional to the sum of the areas of the two loops
- [B] The amplitude of the maximum net  $emf$  induced due to both the loops is equal to the amplitude of maximum  $emf$  induced in the smaller loop alone
- [C] The net  $emf$  induced due to both the loops is proportional to  $\cos \omega t$
- [D] The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

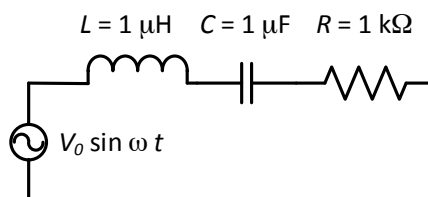
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**Answer for the above question**

**Ans for Q.5: (B) and (D)**

- Q.6 In the circuit shown,  $L = 1 \mu\text{H}$ ,  $C = 1 \mu\text{F}$  and  $R = 1 \text{ k}\Omega$ . They are connected in series with an a.c. source  $V = V_0 \sin \omega t$  as shown. Which of the following options is/are correct?



- [A] The current will be in phase with the voltage if  $\omega = 10^4 \text{ rad.s}^{-1}$
- [B] The frequency at which the current will be in phase with the voltage is independent of  $R$
- [C] At  $\omega \sim 0$  the current flowing through the circuit becomes nearly zero
- [D] At  $\omega \gg 10^6 \text{ rad.s}^{-1}$ , the circuit behaves like a capacitor
- Q.7 For an isosceles prism of angle  $A$  and refractive index  $\mu$ , it is found that the angle of minimum deviation  $\delta_m = A$ . Which of the following options is/are correct?
- [A] For the angle of incidence  $i_1 = A$ , the ray inside the prism is parallel to the base of the prism
- [B] For this prism, the refractive index  $\mu$  and the angle of prism  $A$  are related as  $A = \frac{1}{2} \cos^{-1} \left( \frac{\mu}{2} \right)$
- [C] At minimum deviation, the incident angle  $i_1$  and the refracting angle  $r_1$  at the first refracting surface are related by  $r_1 = (i_1/2)$
- [D] For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is  $i_1 = \sin^{-1} \left[ \sin A \sqrt{4 \cos^2 \frac{A}{2} - 1} - \cos A \right]$

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### Answers for the above questions

<b>Ans for Q.6: (B) and (C)</b>	<b>Ans for Q.7: (A), (C) and (D)</b>
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**SECTION 2 (Maximum Marks: 15)**

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct answer is darkened  
*Zero Marks* : 0 In all other cases

- Q.8 A drop of liquid of radius  $R = 10^{-2}$  m having surface tension  $S = \frac{0.1}{4\pi} \text{ Nm}^{-1}$  divides itself into  $K$  identical drops. In this process the total change in the surface energy  $\Delta U = 10^{-3}$  J. If  $K = 10^\alpha$  then the value of  $\alpha$  is
- Q.9 An electron in a hydrogen atom undergoes a transition from an orbit with quantum number  $n_i$  to another with quantum number  $n_f$ .  $V_i$  and  $V_f$  are respectively the initial and final potential energies of the electron. If  $\frac{V_i}{V_f} = 6.25$ , then the *smallest possible*  $n_f$  is

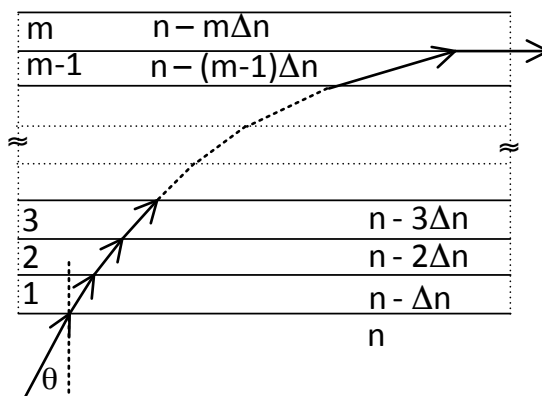
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**Answers for the above questions**

<b>Ans for Q.8: (6)</b>	<b>Ans for Q.9: (5)</b>
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- Q.10 A monochromatic light is travelling in a medium of refractive index  $n = 1.6$ . It enters a stack of glass layers from the bottom side at an angle  $\theta = 30^\circ$ . The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as  $n_m = n - m\Delta n$ , where  $n_m$  is the refractive index of the  $m^{\text{th}}$  slab and  $\Delta n = 0.1$  (see the figure). The ray is refracted out parallel to the interface between the  $(m - 1)^{\text{th}}$  and  $m^{\text{th}}$  slabs from the right side of the stack. What is the value of  $m$ ?



Space for rough work

Answer for the above question

**Ans for Q.10: (8)**



- Q.11 A stationary source emits sound of frequency  $f_0 = 492$  Hz. The sound is *reflected* by a large car *approaching* the source with a speed of  $2 \text{ ms}^{-1}$ . The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is  $330 \text{ ms}^{-1}$  and the car reflects the sound at the frequency *it* has received).
- Q.12  $^{131}\text{I}$  is an isotope of Iodine that  $\beta$  decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with  $^{131}\text{I}$  is injected into the blood of a person. The activity of the amount of  $^{131}\text{I}$  injected was  $2.4 \times 10^5$  Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use  $e^x \approx 1 + x$  for  $|x| \ll 1$  and  $\ln 2 \approx 0.7$ ).

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**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.11: (6)</b>	<b>Ans for Q.12: (5)</b>
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**SECTION 3 (Maximum Marks: 18)**

- This section contains **SIX** questions of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

<i>Full Marks</i>	: +3	If only the bubble corresponding to the correct option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks:</i>	-1	In all other cases

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**Space for rough work**

**Answer Q.13, Q.14 and Q.15 by appropriately matching the information given in the three columns of the following table.**

A charged particle (electron or proton) is introduced at the origin ( $x = 0, y = 0, z = 0$ ) with a given initial velocity $\vec{v}$ . A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$ , electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities $E_0, B_0$ are positive in magnitude.		
Column 1	Column 2	Column 3
(I) Electron with $\vec{v} = 2 \frac{E_0}{B_0} \hat{x}$	(i) $\vec{E} = E_0 \hat{z}$	(P) $\vec{B} = -B_0 \hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0} \hat{y}$	(ii) $\vec{E} = -E_0 \hat{y}$	(Q) $\vec{B} = B_0 \hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0 \hat{x}$	(R) $\vec{B} = B_0 \hat{y}$
(IV) Proton with $\vec{v} = 2 \frac{E_0}{B_0} \hat{x}$	(iv) $\vec{E} = E_0 \hat{x}$	(S) $\vec{B} = B_0 \hat{z}$

Q.13 In which case will the particle move in a straight line with *constant* velocity?

- [A] (III) (ii) (R)      [B] (IV) (i) (S)      [C] (III) (iii) (P)      [D] (II) (iii) (S)

Q.14 In which case will the particle describe a helical path with axis along the positive  $z$  direction?

- [A] (IV) (i) (S)      [B] (II) (ii) (R)      [C] (III) (iii) (P)      [D] (IV) (ii) (R)

Q.15 In which case would the particle move in a straight line along the negative direction of  $y$ -axis (*i.e.*, move along  $-\hat{y}$ )?

- [A] (II) (iii) (Q)      [B] (III) (ii) (R)      [C] (IV) (ii) (S)      [D] (III) (ii) (P)

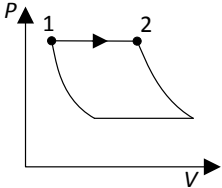
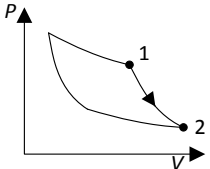
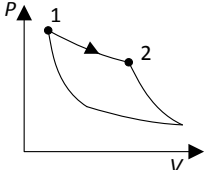
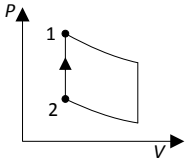
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### Answers for the above questions

<b>Ans for Q.13: (D)</b>	<b>Ans for Q.14: (A)</b>	<b>Ans for Q.15: (B)</b>
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**Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.**

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding $P - V$ diagrams in column 3 of the table. Consider only the path from state 1 to state 2. $W$ denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here $\gamma$ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is $n$ .		
Column 1	Column 2	Column 3
(I) $W_{1 \rightarrow 2} = \frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1)$	(i)  Isothermal	(P) 
(II) $W_{1 \rightarrow 2} = -PV_2 + PV_1$	(ii)  Isochoric	(Q) 
(III) $W_{1 \rightarrow 2} = 0$	(iii)  Isobaric	(R) 
(IV) $W_{1 \rightarrow 2} = -nRT \ln\left(\frac{V_2}{V_1}\right)$	(iv)  Adiabatic	(S) 

**Space for rough work**

Q.16 Which of the following options is the only correct representation of a process in which  $\Delta U = \Delta Q - P\Delta V$ ?

- [A] (II) (iv) (R)      [B] (III) (iii) (P)      [C] (II) (iii) (S)      [D] (II) (iii) (P)

Q.17 Which one of the following options is the correct combination?

- [A] (IV) (ii) (S)      [B] (III) (ii) (S)      [C] (II) (iv) (P)      [D] (II) (iv) (R)

Q.18 Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

- [A] (I) (ii) (Q)      [B] (IV) (ii) (R)      [C] (III) (iv) (R)      [D] (I) (iv) (Q)

**END OF PART I : PHYSICS**

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**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.16: (D)</b>	<b>Ans for Q.17: (B)</b>	<b>Ans for Q.18: (D)</b>
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## PART II : CHEMISTRY

### SECTION 1 (Maximum Marks: 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

<i>Full Marks</i>	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
<i>Partial Marks</i>	: +1	For darkening a bubble corresponding <b>to each correct option</b> , provided NO incorrect option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks</i>	: -2	In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.19 An ideal gas is expanded from  $(p_1, V_1, T_1)$  to  $(p_2, V_2, T_2)$  under different conditions. The correct statement(s) among the following is(are)

- [A] The work done on the gas is maximum when it is compressed irreversibly from  $(p_2, V_2)$  to  $(p_1, V_1)$  against constant pressure  $p_1$
- [B] If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic
- [C] The work done by the gas is less when it is expanded reversibly from  $V_1$  to  $V_2$  under adiabatic conditions as compared to that when expanded reversibly from  $V_1$  to  $V_2$  under isothermal conditions
- [D] The change in internal energy of the gas is (i) zero, if it is expanded reversibly with  $T_1 = T_2$ , and (ii) positive, if it is expanded reversibly under adiabatic conditions with  $T_1 \neq T_2$

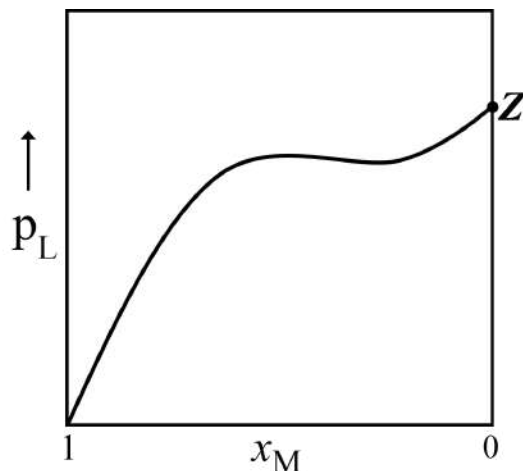
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**Answer for the above question**

**Ans for Q.19: (A), (B), and (C)**

Q.20 For a solution formed by mixing liquids **L** and **M**, the vapour pressure of **L** plotted against the mole fraction of **M** in solution is shown in the following figure. Here  $x_L$  and  $x_M$  represent mole fractions of **L** and **M**, respectively, in the solution. The correct statement(s) applicable to this system is(are)



- [A] The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed from  $x_L = 0$  to  $x_L = 1$
- [B] The point **Z** represents vapour pressure of pure liquid **L** and Raoult's law is obeyed when  $x_L \rightarrow 1$
- [C] The point **Z** represents vapour pressure of pure liquid **M** and Raoult's law is obeyed when  $x_L \rightarrow 0$
- [D] Attractive intermolecular interactions between **L-L** in pure liquid **L** and **M-M** in pure liquid **M** are stronger than those between **L-M** when mixed in solution

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Space for rough work

**Answer for the above question**

**Ans for Q.20: (B) and (D)**

Q.21 The correct statement(s) about the oxoacids,  $\text{HClO}_4$  and  $\text{HClO}$ , is(are)

- [A] The central atom in both  $\text{HClO}_4$  and  $\text{HClO}$  is  $sp^3$  hybridized
- [B]  $\text{HClO}_4$  is more acidic than  $\text{HClO}$  because of the resonance stabilization of its anion
- [C]  $\text{HClO}_4$  is formed in the reaction between  $\text{Cl}_2$  and  $\text{H}_2\text{O}$
- [D] The conjugate base of  $\text{HClO}_4$  is weaker base than  $\text{H}_2\text{O}$

Q.22 The colour of the  $\text{X}_2$  molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to

- [A] the physical state of  $\text{X}_2$  at room temperature changes from gas to solid down the group
- [B] decrease in ionization energy down the group
- [C] decrease in  $\pi^*-\sigma^*$  gap down the group
- [D] decrease in HOMO-LUMO gap down the group

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**Space for rough work**

**Answers for the above questions**

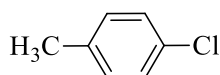
<b>Ans for Q.21: (A), (B), and (D)</b>	<b>Ans for Q.22: (C) and (D)</b>
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Q.23 Addition of excess aqueous ammonia to a pink coloured aqueous solution of  $\text{MCl}_2 \cdot 6\text{H}_2\text{O}$  (**X**) and  $\text{NH}_4\text{Cl}$  gives an octahedral complex **Y** in the presence of air. In aqueous solution, complex **Y** behaves as 1:3 electrolyte. The reaction of **X** with excess  $\text{HCl}$  at room temperature results in the formation of a blue coloured complex **Z**. The calculated spin only magnetic moment of **X** and **Z** is 3.87 B.M., whereas it is zero for complex **Y**. Among the following options, which statement(s) is(are) correct?

- [A] Addition of silver nitrate to **Y** gives only two equivalents of silver chloride
- [B] The hybridization of the central metal ion in **Y** is  $d^2sp^3$
- [C] **Z** is a tetrahedral complex
- [D] When **X** and **Z** are in equilibrium at  $0^\circ\text{C}$ , the colour of the solution is pink

Q.24 The IUPAC name(s) of the following compound is(are)



- [A] 1-chloro-4-methylbenzene
- [B] 4-chlorotoluene
- [C] 4-methylchlorobenzene
- [D] 1-methyl-4-chlorobenzene

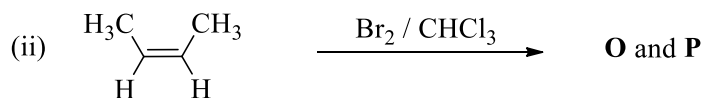
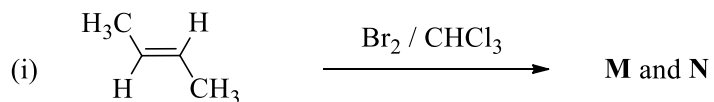
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### Answers for the above questions

<b>Ans for Q.23: (B), (C), and (D)</b>	<b>Ans for Q.24: (A) and (B)</b>
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Q.25 The correct statement(s) for the following addition reactions is(are)



- [A] **O** and **P** are identical molecules  
 [B] (**M** and **O**) and (**N** and **P**) are two pairs of diastereomers  
 [C] (**M** and **O**) and (**N** and **P**) are two pairs of enantiomers  
 [D] Bromination proceeds through *trans*-addition in both the reactions

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Space for rough work

**Answer for the above question**

**Ans for Q.25: (B) and (D)**

**SECTION 2 (Maximum Marks: 15)**

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct answer is darkened  
*Zero Marks* : 0 In all other cases

Q.26 A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 400 pm. If the density of the substance in the crystal is  $8 \text{ g cm}^{-3}$ , then the number of atoms present in 256 g of the crystal is  $N \times 10^{24}$ . The value of  $N$  is

Q.27 The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of  $1 \text{ cm}^2$ . The conductance of this solution was found to be  $5 \times 10^{-7} \text{ S}$ . The pH of the solution is 4. The value of limiting molar conductivity ( $\Lambda_m^\circ$ ) of this weak monobasic acid in aqueous solution is  $Z \times 10^2 \text{ S cm}^{-1} \text{ mol}^{-1}$ . The value of  $Z$  is

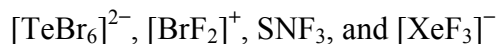
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**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.26: (2)</b>	<b>Ans for Q.27: (6)</b>
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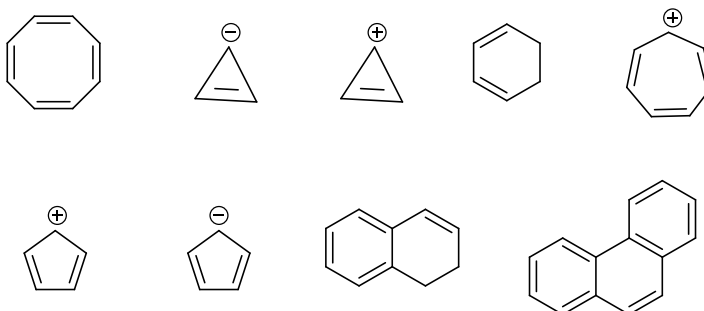
Q.28 The sum of the number of lone pairs of electrons on each central atom in the following species is



(Atomic numbers: N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54)

Q.29 Among  $\text{H}_2$ ,  $\text{He}_2^+$ ,  $\text{Li}_2$ ,  $\text{Be}_2$ ,  $\text{B}_2$ ,  $\text{C}_2$ ,  $\text{N}_2$ ,  $\text{O}_2^-$ , and  $\text{F}_2$ , the number of diamagnetic species is  
(Atomic numbers: H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, F = 9)

Q.30 Among the following, the number of aromatic compound(s) is




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Space for rough work

### Answers for the above questions

**Ans for Q.28: (6)**

**Ans for Q.30: (5)**

**Due to printing inconsistency in Q.29 in Code1 of Paper1 (and the corresponding question in all other codes), +3 marks will be awarded to all candidates for this question.**

**SECTION 3 (Maximum Marks: 18)**

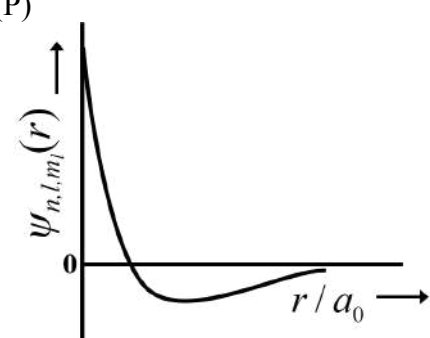
- This section contains **SIX** questions of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

<i>Full Marks</i>	: +3	If only the bubble corresponding to the correct option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks:</i>	-1	In all other cases

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**Space for rough work**

**Answer Q.31, Q.32 and Q.33 by appropriately matching the information given in the three columns of the following table.**

The wave function, $\psi_{n,l,m_l}$ is a mathematical function whose value depends upon spherical polar coordinates $(r, \theta, \phi)$ of the electron and characterized by the quantum numbers $n$ , $l$ and $m_l$ . Here $r$ is distance from nucleus, $\theta$ is colatitude and $\phi$ is azimuth. In the mathematical functions given in the Table, $Z$ is atomic number and $a_o$ is Bohr radius.		
Column 1	Column 2	Column 3
(I) 1s orbital	(i) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_o}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_o}\right)}$	(P) 
(II) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_o^3}$
(III) 2p <sub>z</sub> orbital	(iii) $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_o}\right)^{\frac{5}{2}} r e^{-\left(\frac{Zr}{2a_o}\right)} \cos\theta$	(R) Probability density is maximum at nucleus
(IV) 3d <sub>z<sup>2</sup></sub> orbital	(iv) xy-plane is a nodal plane	(S) Energy needed to excite electron from $n = 2$ state to $n = 4$ state is $\frac{27}{32}$ times the energy needed to excite electron from $n = 2$ state to $n = 6$ state

Q.31 For the given orbital in Column 1, the only CORRECT combination for any hydrogen-like species is

- [A] (I) (ii) (S)      [B] (IV) (iv) (R)      [C] (II) (ii) (P)      [D] (III) (iii) (P)

Q.32 For hydrogen atom, the only CORRECT combination is

- [A] (I) (i) (S)      [B] (II) (i) (Q)      [C] (I) (i) (P)      [D] (I) (iv) (R)

Q.33 For He<sup>+</sup> ion, the only INCORRECT combination is

- [A] (I) (i) (R)      [B] (II) (ii) (Q)      [C] (I) (iii) (R)      [D] (I) (i) (S)

**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.31: (C)</b>	<b>Ans for Q.32: (A)</b>	<b>Ans for Q.33: (C)</b>
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**Answer Q.34, Q.35 and Q.36 by appropriately matching the information given in the three columns of the following table.**

Columns 1, 2 and 3 contain starting materials, reaction conditions, and type of reactions, respectively.		
Column 1	Column 2	Column 3
(I) Toluene	(i) NaOH/ Br <sub>2</sub>	(P) Condensation
(II) Acetophenone	(ii) Br <sub>2</sub> / hν	(Q) Carboxylation
(III) Benzaldehyde	(iii) (CH <sub>3</sub> CO) <sub>2</sub> O/ CH <sub>3</sub> COOK	(R) Substitution
(IV) Phenol	(iv) NaOH/ CO <sub>2</sub>	(S) Haloform

Q.34 For the synthesis of benzoic acid, the only CORRECT combination is

- [A] (II) (i) (S)      [B] (IV) (ii) (P)      [C] (I) (iv) (Q)      [D] (III) (iv) (R)

Q.35 The only CORRECT combination that gives two different carboxylic acids is

- [A] (II) (iv) (R)      [B] (IV) (iii) (Q)      [C] (III) (iii) (P)      [D] (I) (i) (S)

Q.36 The only CORRECT combination in which the reaction proceeds through radical mechanism is

- [A] (III) (ii) (P)      [B] (IV) (i) (Q)      [C] (II) (iii) (R)      [D] (I) (ii) (R)

**END OF PART II : CHEMISTRY**

Space for rough work

**Answers for the above questions**

<b>Ans for Q.34: (A)</b>	<b>Ans for Q.35: (C)</b>	<b>Ans for Q.36: (D)</b>
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## PART III : MATHEMATICS

### SECTION 1 (Maximum Marks: 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

<i>Full Marks</i>	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
<i>Partial Marks</i>	: +1	For darkening a bubble corresponding <b>to each correct option</b> , provided NO incorrect option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks</i>	: -2	In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.37 If  $2x - y + 1 = 0$  is a tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ , then which of the following CANNOT be sides of a right angled triangle?

- [A]  $a, 4, 1$                       [B]  $a, 4, 2$                       [C]  $2a, 8, 1$                       [D]  $2a, 4, 1$

Q.38 If a chord, which is not a tangent, of the parabola  $y^2 = 16x$  has the equation  $2x + y = p$ , and midpoint  $(h, k)$ , then which of the following is(are) possible value(s) of  $p, h$  and  $k$ ?

- [A]  $p = -2, h = 2, k = -4$                       [B]  $p = -1, h = 1, k = -3$   
[C]  $p = 2, h = 3, k = -4$                       [D]  $p = 5, h = 4, k = -3$

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Space for rough work

### Answers for the above questions

<b>Ans for Q.37: (A), (B), and (C)</b>	<b>Ans for Q.38: (C)</b>
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Q.42 Let  $a, b, x$  and  $y$  be real numbers such that  $a - b = 1$  and  $y \neq 0$ . If the complex number  $z = x + iy$  satisfies  $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$ , then which of the following is(are) possible value(s) of  $x$ ?

[A]  $-1 + \sqrt{1 - y^2}$

[B]  $-1 - \sqrt{1 - y^2}$

[C]  $1 + \sqrt{1 + y^2}$

[D]  $1 - \sqrt{1 + y^2}$

Q.43 Let  $X$  and  $Y$  be two events such that  $P(X) = \frac{1}{3}$ ,  $P(X|Y) = \frac{1}{2}$  and  $P(Y|X) = \frac{2}{5}$ . Then

[A]  $P(Y) = \frac{4}{15}$

[B]  $P(X'|Y) = \frac{1}{2}$

[C]  $P(X \cap Y) = \frac{1}{5}$

[D]  $P(X \cup Y) = \frac{2}{5}$

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.42: (A) and (B)</b>	<b>Ans for Q.43: (A) and (B)</b>
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**SECTION 2 (Maximum Marks: 15)**

- This section contains **FIVE** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct answer is darkened  
*Zero Marks* : 0 In all other cases

Q.44 For how many values of  $p$ , the circle  $x^2 + y^2 + 2x + 4y - p = 0$  and the coordinate axes have exactly three common points?

Q.45 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f(0) = 0$ ,  $f\left(\frac{\pi}{2}\right) = 3$  and  $f'(0) = 1$ . If

$$g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt$$

for  $x \in (0, \frac{\pi}{2}]$ , then  $\lim_{x \rightarrow 0} g(x) =$

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**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.44: (2)</b>	<b>Ans for Q.45: (2)</b>
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Q.46 For a real number  $\alpha$ , if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then  $1 + \alpha + \alpha^2 =$

Q.47 Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let  $x$  be the number of such words where no letter is repeated; and let  $y$  be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then,  $\frac{y}{9x} =$

Q.48 The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

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**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.46: (1)</b>	<b>Ans for Q.47: (5)</b>	<b>Ans for Q.48: (6)</b>
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**SECTION 3 (Maximum Marks: 18)**

- This section contains **SIX** questions of matching type
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

<i>Full Marks</i>	: +3	If only the bubble corresponding to the correct option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks:</i>	-1	In all other cases

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**Space for rough work**

**Answer Q.49, Q.50 and Q.51 by appropriately matching the information given in the three columns of the following table.**

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.		
Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2 + 1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2 - 1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2 + 1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

Q.49 For  $a = \sqrt{2}$ , if a tangent is drawn to a suitable conic (Column 1) at the point of contact  $(-1, 1)$ , then which of the following options is the only CORRECT combination for obtaining its equation?

- [A] (I) (i) (P)      [B] (I) (ii) (Q)      [C] (II) (ii) (Q)      [D] (III) (i) (P)

Q.50 If a tangent to a suitable conic (Column 1) is found to be  $y = x + 8$  and its point of contact is  $(8, 16)$ , then which of the following options is the only CORRECT combination?

- [A] (I) (ii) (Q)      [B] (II) (iv) (R)      [C] (III) (i) (P)      [D] (III) (ii) (Q)

Q.51 The tangent to a suitable conic (Column 1) at  $(\sqrt{3}, \frac{1}{2})$  is found to be  $\sqrt{3}x + 2y = 4$ , then which of the following options is the only CORRECT combination?

- [A] (IV) (iii) (S)      [B] (IV) (iv) (S)      [C] (II) (iii) (R)      [D] (II) (iv) (R)

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**Space for rough work**

### Answers for the above questions

<b>Ans for Q.49: (B)</b>	<b>Ans for Q.50: (C)</b>	<b>Ans for Q.51: (D)</b>
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**Answer Q.52, Q.53 and Q.54 by appropriately matching the information given in the three columns of the following table.**

Let $f(x) = x + \log_e x - x \log_e x$ , $x \in (0, \infty)$ . • Column 1 contains information about zeros of $f(x)$ , $f'(x)$ and $f''(x)$ . • Column 2 contains information about the limiting behavior of $f(x)$ , $f'(x)$ and $f''(x)$ at infinity. • Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$ .		
Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x \rightarrow \infty} f(x) = 0$	(P) $f$ is increasing in $(0, 1)$
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x \rightarrow \infty} f(x) = -\infty$	(Q) $f$ is decreasing in $(e, e^2)$
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x \rightarrow \infty} f'(x) = -\infty$	(R) $f'$ is increasing in $(0, 1)$
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x \rightarrow \infty} f''(x) = 0$	(S) $f'$ is decreasing in $(e, e^2)$

Q.52 Which of the following options is the only CORRECT combination?

- [A] (I) (i) (P)      [B] (II) (ii) (Q)      [C] (III) (iii) (R)      [D] (IV) (iv) (S)

Q.53 Which of the following options is the only CORRECT combination?

- [A] (I) (ii) (R)      [B] (II) (iii) (S)      [C] (III) (iv) (P)      [D] (IV) (i) (S)

Q.54 Which of the following options is the only INCORRECT combination?

- [A] (I) (iii) (P)      [B] (II) (iv) (Q)      [C] (III) (i) (R)      [D] (II) (iii) (P)

**END OF THE QUESTION PAPER**

Space for rough work

**Answers for the above questions**

<b>Ans for Q.52: (B)</b>	<b>Ans for Q.53: (B)</b>	<b>Ans for Q.54: (C)</b>
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**SPACE FOR ROUGH WORK**



## PART I : PHYSICS

### SECTION 1 (Maximum Marks: 21)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened

*Zero Marks* : 0 If none of the bubbles is darkened

*Negative Marks*: -1 In all other cases

Q.1 Consider an expanding sphere of instantaneous radius  $R$  whose total mass remains constant. The expansion is such that the *instantaneous* density  $\rho$  remains uniform throughout the volume. The rate of fractional change in density  $\left(\frac{1}{\rho} \frac{d\rho}{dt}\right)$  is constant. The velocity  $v$  of any point on the surface of the expanding sphere is proportional to

[A]  $R$

[B]  $R^3$

[C]  $\frac{1}{R}$

[D]  $R^{2/3}$

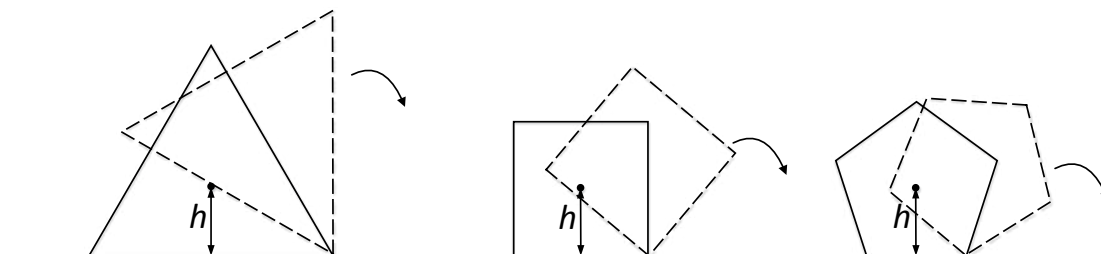
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Space for rough work

**Answer for the above question**

**Ans for Q.1: (A)**

- Q.2 Consider regular polygons with number of sides  $n = 3, 4, 5, \dots$  as shown in the figure. The center of mass of all the polygons is at height  $h$  from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is  $\Delta$ . Then  $\Delta$  depends on  $n$  and  $h$  as



[A]  $\Delta = h \sin^2 \left( \frac{\pi}{n} \right)$

[B]  $\Delta = h \left( \frac{1}{\cos \left( \frac{\pi}{n} \right)} - 1 \right)$

[C]  $\Delta = h \sin \left( \frac{2\pi}{n} \right)$

[D]  $\Delta = h \tan^2 \left( \frac{\pi}{2n} \right)$

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Space for rough work

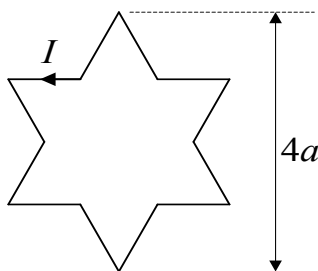
**Answer for the above question**

**Ans for Q.2: (B)**

Q.3 A photoelectric material having work-function  $\phi_0$  is illuminated with light of wavelength  $\lambda$  ( $\lambda < \frac{hc}{\phi_0}$ ). The fastest photoelectron has a de Broglie wavelength  $\lambda_d$ . A change in wavelength of the incident light by  $\Delta\lambda$  results in a change  $\Delta\lambda_d$  in  $\lambda_d$ . Then the ratio  $\Delta\lambda_d/\Delta\lambda$  is proportional to

- [A]  $\lambda_d/\lambda$  [B]  $\lambda_d^2/\lambda^2$  [C]  $\lambda_d^3/\lambda$  [D]  $\lambda_d^3/\lambda^2$

Q.4 A symmetric star shaped conducting wire loop is carrying a steady state current  $I$  as shown in the figure. The distance between the diametrically opposite vertices of the star is  $4a$ . The magnitude of the magnetic field at the center of the loop is



- [A]  $\frac{\mu_0 I}{4\pi a} 6[\sqrt{3} - 1]$  [B]  $\frac{\mu_0 I}{4\pi a} 6[\sqrt{3} + 1]$   
[C]  $\frac{\mu_0 I}{4\pi a} 3[\sqrt{3} - 1]$  [D]  $\frac{\mu_0 I}{4\pi a} 3[2 - \sqrt{3}]$

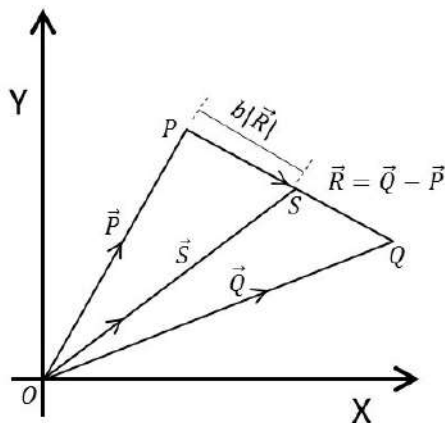
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Space for rough work

### Answers for the above questions

<b>Ans for Q.3: (D)</b>	<b>Ans for Q.4: (A)</b>
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- Q.5 Three vectors  $\vec{P}$ ,  $\vec{Q}$  and  $\vec{R}$  are shown in the figure. Let  $S$  be any point on the vector  $\vec{R}$ . The distance between the points  $P$  and  $S$  is  $b|\vec{R}|$ . The general relation among vectors  $\vec{P}$ ,  $\vec{Q}$  and  $\vec{S}$  is



- [A]  $\vec{S} = (1 - b)\vec{P} + b\vec{Q}$                       [B]  $\vec{S} = (b - 1)\vec{P} + b\vec{Q}$   
[C]  $\vec{S} = (1 - b^2)\vec{P} + b\vec{Q}$                       [D]  $\vec{S} = (1 - b)\vec{P} + b^2\vec{Q}$

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Space for rough work

**Answer for the above question**

**Ans for Q.5: (A)**

Q.6 A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is  $3 \times 10^5$  times heavier than the Earth and is at a distance  $2.5 \times 10^4$  times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is  $v_e = 11.2 \text{ km s}^{-1}$ . The minimum initial velocity ( $v_s$ ) required for the rocket to be able to leave the *Sun-Earth system* is closest to  
(Ignore the rotation and revolution of the Earth and the presence of any other planet)

[A]  $v_s = 22 \text{ km s}^{-1}$

[B]  $v_s = 42 \text{ km s}^{-1}$

[C]  $v_s = 62 \text{ km s}^{-1}$

[D]  $v_s = 72 \text{ km s}^{-1}$

Q.7 A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is  $\delta T = 0.01$  seconds and he measures the depth of the well to be  $L = 20$  meters. Take the acceleration due to gravity  $g = 10 \text{ ms}^{-2}$  and the velocity of sound is  $300 \text{ ms}^{-1}$ . Then the fractional error in the measurement,  $\delta L/L$ , is closest to

[A] 0.2%

[B] 1%

[C] 3%

[D] 5%

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Space for rough work

### Answers for the above questions

Ans for Q.6: (B)	Ans for Q.7: (B)
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**SECTION 2 (Maximum Marks: 28)**

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

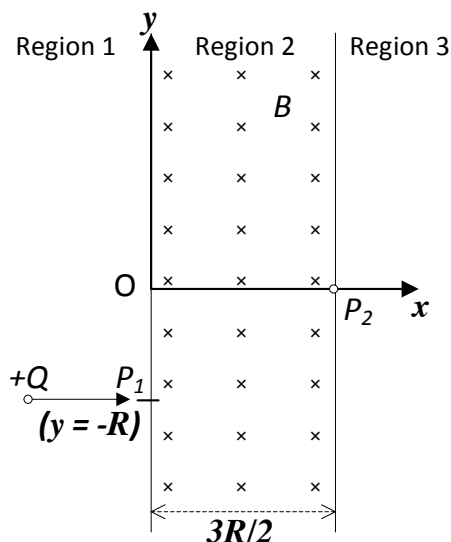
<i>Full Marks</i>	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
<i>Partial Marks</i>	: +1	For darkening a bubble corresponding <b>to each correct option</b> , provided NO incorrect option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks</i>	: -2	In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

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**Space for rough work**

- Q.8 A uniform magnetic field  $B$  exists in the region between  $x = 0$  and  $x = \frac{3R}{2}$  (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge  $+Q$  and momentum  $p$  directed along  $x$ -axis enters region 2 from region 1 at point  $P_1$  ( $y = -R$ ). Which of the following option(s) is/are correct?



- [A] For  $B > \frac{2}{3} \frac{p}{QR}$ , the particle will re-enter region 1
- [B] For  $B = \frac{8}{13} \frac{p}{QR}$ , the particle will enter region 3 through the point  $P_2$  on  $x$ -axis
- [C] When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point  $P_1$  and the farthest point from  $y$ -axis is  $p/\sqrt{2}$
- [D] For a fixed  $B$ , particles of same charge  $Q$  and same velocity  $v$ , the distance between the point  $P_1$  and the point of re-entry into region 1 is inversely proportional to the mass of the particle

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**Answer for the above question**

**Ans for Q.8: (A) and (B)**

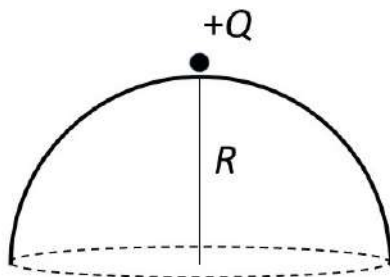
Q.9 The instantaneous voltages at three terminals marked X, Y and Z are given by

$$\begin{aligned} V_X &= V_0 \sin \omega t, \\ V_Y &= V_0 \sin \left( \omega t + \frac{2\pi}{3} \right) \text{ and} \\ V_Z &= V_0 \sin \left( \omega t + \frac{4\pi}{3} \right). \end{aligned}$$

An ideal voltmeter is configured to read *rms* value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

- [A]  $V_{XY}^{rms} = V_0 \sqrt{\frac{3}{2}}$   
 [B]  $V_{YZ}^{rms} = V_0 \sqrt{\frac{1}{2}}$   
 [C]  $V_{XY}^{rms} = V_0$   
 [D] independent of the choice of the two terminals

Q.10 A point charge  $+Q$  is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



- [A] The electric flux passing through the *curved* surface of the hemisphere is  $-\frac{Q}{2\epsilon_0} \left( 1 - \frac{1}{\sqrt{2}} \right)$   
 [B] Total flux through the curved and the flat surfaces is  $\frac{Q}{\epsilon_0}$   
 [C] The component of the electric field normal to the flat surface is constant over the surface  
 [D] The circumference of the flat surface is an equipotential

Space for rough work

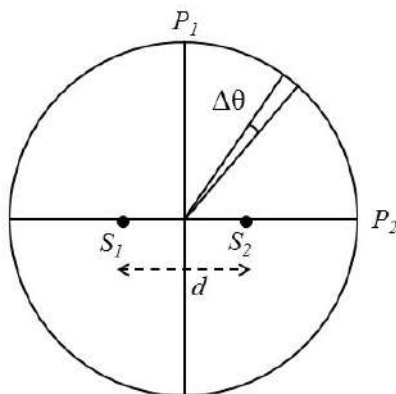
### Answers for the above questions

**Ans for Q.9: (A) and (D)**

**Ans for Q.10: (A) and (D)**



- Q.11 Two coherent monochromatic point sources  $S_1$  and  $S_2$  of wavelength  $\lambda = 600 \text{ nm}$  are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance  $d = 1.8 \text{ mm}$ . This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is  $\Delta\theta$ . Which of the following options is/are correct?



- [A] A dark spot will be formed at the point  $P_2$
- [B] At  $P_2$  the order of the fringe will be maximum
- [C] The total number of fringes produced between  $P_1$  and  $P_2$  in the first quadrant is close to 3000
- [D] The angular separation between two consecutive bright spots decreases as we move from  $P_1$  to  $P_2$  along the first quadrant

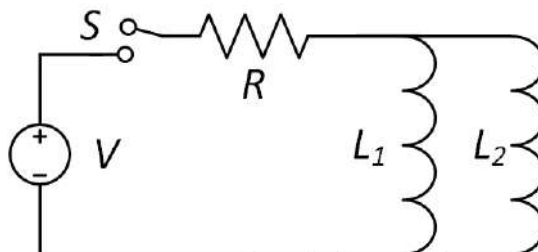
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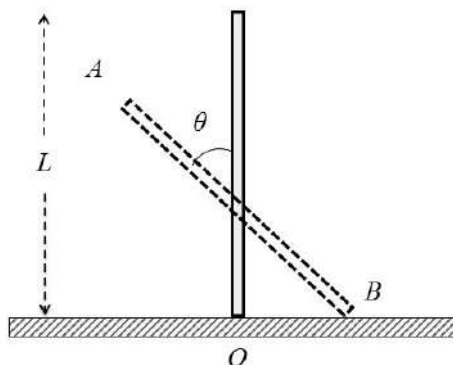
**Answer for the above question**

**Ans for Q.11: (B) and (C)**

- Q.12 A source of constant voltage  $V$  is connected to a resistance  $R$  and two ideal inductors  $L_1$  and  $L_2$  through a switch  $S$  as shown. There is no mutual inductance between the two inductors. The switch  $S$  is initially open. At  $t = 0$ , the switch is closed and current begins to flow. Which of the following options is/are correct?



- [A] After a long time, the current through  $L_1$  will be  $\frac{V}{R} \frac{L_2}{L_1 + L_2}$
- [B] After a long time, the current through  $L_2$  will be  $\frac{V}{R} \frac{L_1}{L_1 + L_2}$
- [C] The ratio of the currents through  $L_1$  and  $L_2$  is fixed at all times ( $t > 0$ )
- [D] At  $t = 0$ , the current through the resistance  $R$  is  $\frac{V}{R}$
- Q.13 A rigid uniform bar AB of length  $L$  is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is  $\theta$ . Which of the following statements about its motion is/are correct?



- [A] The midpoint of the bar will fall vertically downward
- [B] The trajectory of the point A is a parabola
- [C] Instantaneous torque about the point in contact with the floor is proportional to  $\sin \theta$
- [D] When the bar makes an angle  $\theta$  with the vertical, the displacement of its midpoint from the initial position is proportional to  $(1 - \cos \theta)$

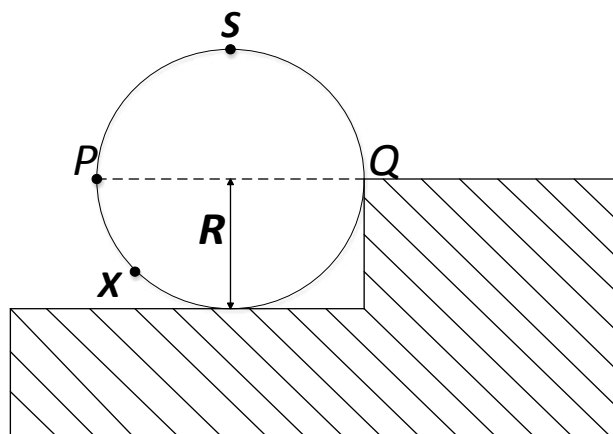
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### Answers for the above questions

**Ans for Q.12: (A), (B), and (C)**

**Ans for Q.13: (A), (C), and (D)**

- Q.14 A wheel of radius  $R$  and mass  $M$  is placed at the bottom of a fixed step of height  $R$  as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque  $\tau$  about an axis normal to the plane of the paper passing through the point  $Q$ . Which of the following options is/are correct?



- [A] If the force is applied at point  $P$  tangentially then  $\tau$  decreases continuously as the wheel climbs
- [B] If the force is applied normal to the circumference at point  $X$  then  $\tau$  is constant
- [C] If the force is applied normal to the circumference at point  $P$  then  $\tau$  is zero
- [D] If the force is applied tangentially at point  $S$  then  $\tau \neq 0$  but the wheel never climbs the step

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Space for rough work

**Answer for the above question**

**Ans for Q.14: {(C)} or {(C) and (D)}**

**SECTION 3 (Maximum Marks: 12)**

- This section contains **TWO** paragraphs
- Based on each paragraph, there are **TWO** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened  
*Zero Marks* : 0 In all other cases

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**Space for rough work**

### PARAGRAPH 1

Consider a simple  $RC$  circuit as shown in Figure 1.

Process 1: In the circuit the switch  $S$  is closed at  $t = 0$  and the capacitor is fully charged to voltage  $V_0$  (i.e., charging continues for time  $T \gg RC$ ). In the process some dissipation ( $E_D$ ) occurs across the resistance  $R$ . The amount of energy finally stored in the fully charged capacitor is  $E_C$ .

Process 2: In a different process the voltage is first set to  $\frac{V_0}{3}$  and maintained for a charging time  $T \gg RC$ . Then the voltage is raised to  $\frac{2V_0}{3}$  without discharging the capacitor and again maintained for a time  $T \gg RC$ . The process is repeated one more time by raising the voltage to  $V_0$  and the capacitor is charged to the same final voltage  $V_0$  as in Process 1.

These two processes are depicted in Figure 2.

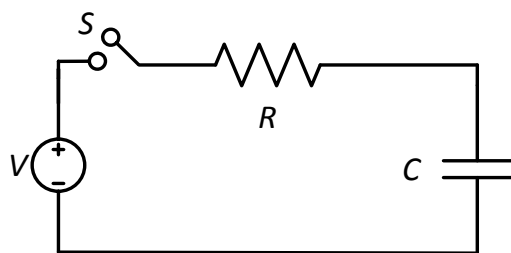


Figure 1

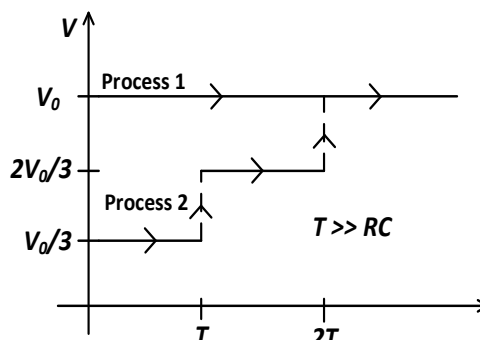


Figure 2

Q.15 In Process 1, the energy stored in the capacitor  $E_C$  and heat dissipated across resistance  $E_D$  are related by:

[A]  $E_C = E_D$

[B]  $E_C = E_D \ln 2$

[C]  $E_C = \frac{1}{2} E_D$

[D]  $E_C = 2E_D$

Q.16 In Process 2, total energy dissipated across the resistance  $E_D$  is:

[A]  $E_D = \frac{1}{2} CV_0^2$

[B]  $E_D = 3 \left( \frac{1}{2} CV_0^2 \right)$

[C]  $E_D = \frac{1}{3} \left( \frac{1}{2} CV_0^2 \right)$

[D]  $E_D = 3 CV_0^2$

### Answers for the above questions

**Ans for Q.15: (A)**

**Ans for Q.16: (C)**

### PARAGRAPH 2

One twirls a circular ring (of mass  $M$  and radius  $R$ ) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is  $r$ . The finger rotates with an angular velocity  $\omega_0$ . The rotating ring *rolls without slipping* on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is  $\mu$  and the acceleration due to gravity is  $g$ .

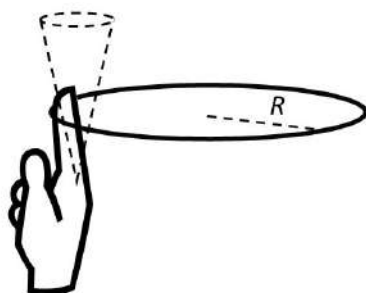


Figure 1

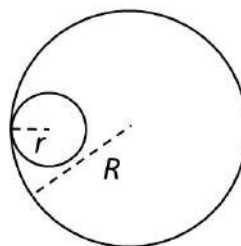


Figure 2

Q.17 The total kinetic energy of the ring is

[A]  $M\omega_0^2 R^2$

[B]  $\frac{1}{2}M\omega_0^2(R-r)^2$

[C]  $M\omega_0^2(R-r)^2$

[D]  $\frac{3}{2}M\omega_0^2(R-r)^2$

Q.18 The minimum value of  $\omega_0$  below which the ring will drop down is

[A]  $\sqrt{\frac{g}{\mu(R-r)}}$

[B]  $\sqrt{\frac{2g}{\mu(R-r)}}$

[C]  $\sqrt{\frac{3g}{2\mu(R-r)}}$

[D]  $\sqrt{\frac{g}{2\mu(R-r)}}$

END OF PART I : PHYSICS

### Answers for the above questions

**Q.17: Due to internal review, all candidates are awarded +3 marks.**

**Ans for Q.18: (A)**

## PART II : CHEMISTRY

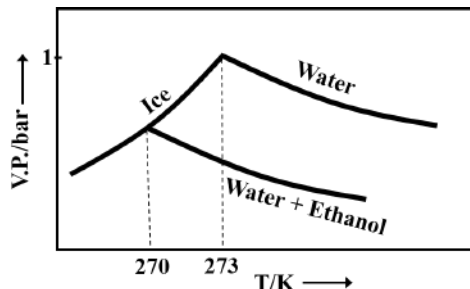
### SECTION 1 (Maximum Marks: 21)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

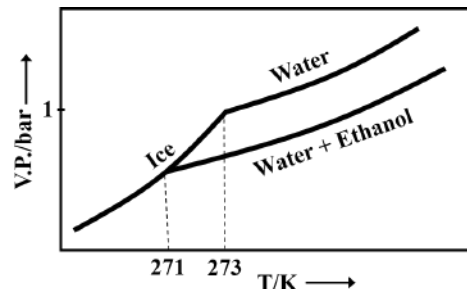
*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened  
*Zero Marks* : 0 If none of the bubbles is darkened  
*Negative Marks*: -1 In all other cases

Q.19 Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as  $2 \text{ K kg mol}^{-1}$ . The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T). [molecular weight of ethanol is  $46 \text{ g mol}^{-1}$ ]  
 Among the following, the option representing change in the freezing point is

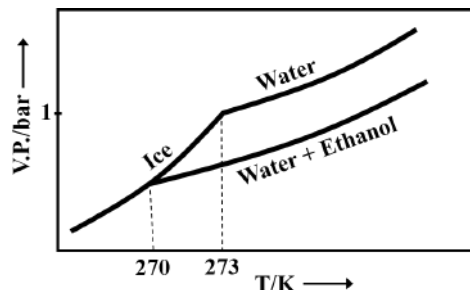
[A]



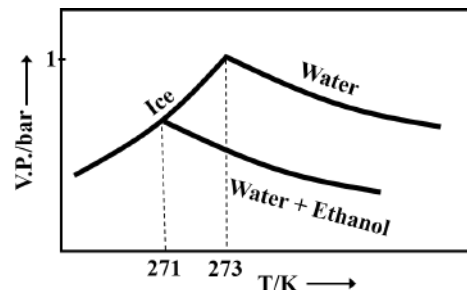
[B]



[C]



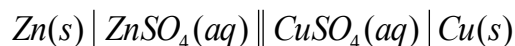
[D]



**Answer for the above question**

**Ans for Q.19: (C)**

Q.20 For the following cell,



when the concentration of  $\text{Zn}^{2+}$  is 10 times the concentration of  $\text{Cu}^{2+}$ , the expression for  $\Delta G$  (in  $\text{J mol}^{-1}$ ) is

[F is Faraday constant; R is gas constant; T is temperature;  $E^\circ(\text{cell}) = 1.1 \text{ V}$ ]

[A]  $1.1F$

[B]  $2.303RT - 2.2F$

[C]  $2.303RT + 1.1F$

[D]  $-2.2F$

Q.21 The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at  $T = 298 \text{ K}$  are

$$\Delta_f G^\circ[\text{C}(\text{graphite})] = 0 \text{ kJ mol}^{-1}$$

$$\Delta_f G^\circ[\text{C}(\text{diamond})] = 2.9 \text{ kJ mol}^{-1}$$

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by  $2 \times 10^{-6} \text{ m}^3 \text{ mol}^{-1}$ . If C(graphite) is converted to C(diamond) isothermally at  $T = 298 \text{ K}$ , the pressure at which C(graphite) is in equilibrium with C(diamond), is

[Useful information:  $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$ ;  $1 \text{ Pa} = 1 \text{ kg m}^{-1} \text{ s}^{-2}$ ;  $1 \text{ bar} = 10^5 \text{ Pa}$ ]

[A] 14501 bar

[B] 58001 bar

[C] 1450 bar

[D] 29001 bar

Q.22 Which of the following combination will produce  $\text{H}_2$  gas?

[A] Fe metal and conc.  $\text{HNO}_3$

[B] Cu metal and conc.  $\text{HNO}_3$

[C] Zn metal and  $\text{NaOH}(\text{aq})$

[D] Au metal and  $\text{NaCN}(\text{aq})$  in the presence of air

Space for rough work

### Answers for the above questions

**Ans for Q.20: (B)**

**Ans for Q.21: (A)**

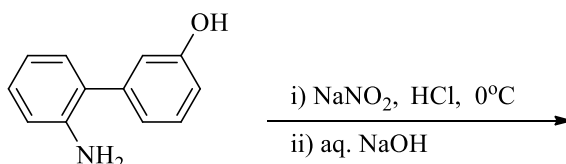
**Ans for Q.22: (C)**



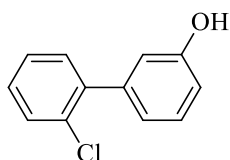
Q.23 The order of the oxidation state of the phosphorus atom in  $\text{H}_3\text{PO}_2$ ,  $\text{H}_3\text{PO}_4$ ,  $\text{H}_3\text{PO}_3$ , and  $\text{H}_4\text{P}_2\text{O}_6$  is

- [A]  $\text{H}_3\text{PO}_3 > \text{H}_3\text{PO}_2 > \text{H}_3\text{PO}_4 > \text{H}_4\text{P}_2\text{O}_6$       [B]  $\text{H}_3\text{PO}_4 > \text{H}_3\text{PO}_2 > \text{H}_3\text{PO}_3 > \text{H}_4\text{P}_2\text{O}_6$   
[C]  $\text{H}_3\text{PO}_4 > \text{H}_4\text{P}_2\text{O}_6 > \text{H}_3\text{PO}_3 > \text{H}_3\text{PO}_2$       [D]  $\text{H}_3\text{PO}_2 > \text{H}_3\text{PO}_3 > \text{H}_4\text{P}_2\text{O}_6 > \text{H}_3\text{PO}_4$

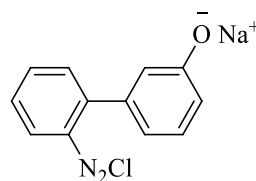
Q.24 The major product of the following reaction is



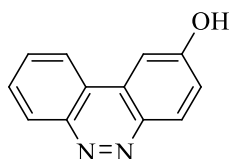
[A]



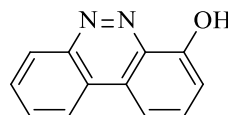
[B]



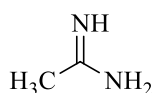
[C]



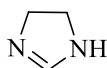
[D]



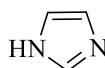
Q.25 The order of basicity among the following compounds is



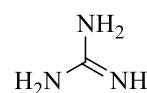
I



II



III



IV

[A]  $\text{II} > \text{I} > \text{IV} > \text{III}$

[B]  $\text{IV} > \text{II} > \text{III} > \text{I}$

[C]  $\text{IV} > \text{I} > \text{II} > \text{III}$

[D]  $\text{I} > \text{IV} > \text{III} > \text{II}$

Space for rough work

**Answers for the above questions**

**Ans for Q.23: (C)**

**Ans for Q.24: (C)**

**Ans for Q.25: (C)**

**SECTION 2 (Maximum Marks: 28)**

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

<i>Full Marks</i>	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
<i>Partial Marks</i>	: +1	For darkening a bubble corresponding <b>to each correct option</b> , provided NO incorrect option is darkened
<i>Zero Marks</i>	: 0	If none of the bubbles is darkened
<i>Negative Marks</i>	: -2	In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.26 The correct statement(s) about surface properties is(are)

- [A] Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system
- [B] The critical temperatures of ethane and nitrogen are 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature
- [C] Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium
- [D] Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution

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**Space for rough work**

**Answer for the above question**

**Ans for Q.26: (A) and (B)**

- Q.27 For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant  $K$  in terms of change in entropy is described by
- [A] With increase in temperature, the value of  $K$  for exothermic reaction decreases because the entropy change of the system is positive
  - [B] With increase in temperature, the value of  $K$  for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases
  - [C] With increase in temperature, the value of  $K$  for endothermic reaction increases because the entropy change of the system is negative
  - [D] With increase in temperature, the value of  $K$  for exothermic reaction decreases because favourable change in entropy of the surroundings decreases
- Q.28 In a bimolecular reaction, the steric factor  $P$  was experimentally determined to be 4.5. The correct option(s) among the following is(are)
- [A] The activation energy of the reaction is unaffected by the value of the steric factor
  - [B] Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation
  - [C] Since  $P = 4.5$ , the reaction will not proceed unless an effective catalyst is used
  - [D] The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

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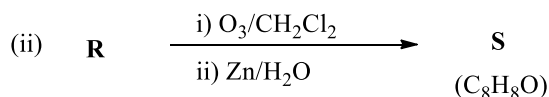
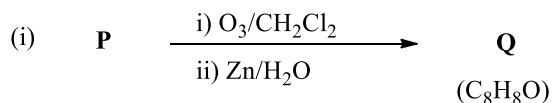
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**Answers for the above questions**

<b>Ans for Q.27: (B) and (D)</b>	<b>Ans for Q.28: (A) and (B)</b>
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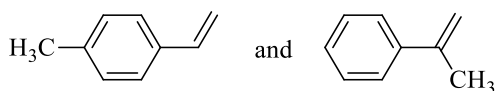


Q.32 Compounds **P** and **R** upon ozonolysis produce **Q** and **S**, respectively. The molecular formula of **Q** and **S** is  $C_8H_8O$ . **Q** undergoes Cannizzaro reaction but not haloform reaction, whereas **S** undergoes haloform reaction but not Cannizzaro reaction.

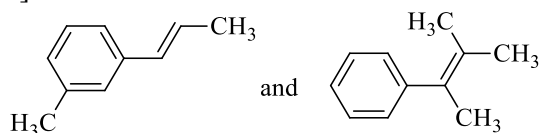


The option(s) with suitable combination of **P** and **R**, respectively, is(are)

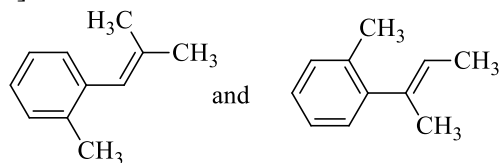
[A]



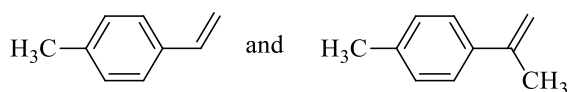
[B]



[C]



[D]




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Space for rough work

**Answer for the above question**

**Ans for Q.32: (A) and (B)**

**SECTION 3 (Maximum Marks: 12)**

- This section contains **TWO** paragraphs
- Based on each paragraph, there are **TWO** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened  
*Zero Marks* : 0 In all other cases

**PARAGRAPH 1**

Upon heating  $\text{KClO}_3$  in the presence of catalytic amount of  $\text{MnO}_2$ , a gas **W** is formed. Excess amount of **W** reacts with white phosphorus to give **X**. The reaction of **X** with pure  $\text{HNO}_3$  gives **Y** and **Z**.

Q.33 **W** and **X** are, respectively

- |  |  |
|--|--|
| [A] $\text{O}_3$ and $\text{P}_4\text{O}_6$    | [B] $\text{O}_2$ and $\text{P}_4\text{O}_6$    |
| [C] $\text{O}_2$ and $\text{P}_4\text{O}_{10}$ | [D] $\text{O}_3$ and $\text{P}_4\text{O}_{10}$ |

Q.34 **Y** and **Z** are, respectively

- |  |  |
|--|--|
| [A] $\text{N}_2\text{O}_3$ and $\text{H}_3\text{PO}_4$ | [B] $\text{N}_2\text{O}_5$ and $\text{HPO}_3$          |
| [C] $\text{N}_2\text{O}_4$ and $\text{HPO}_3$          | [D] $\text{N}_2\text{O}_4$ and $\text{H}_3\text{PO}_3$ |

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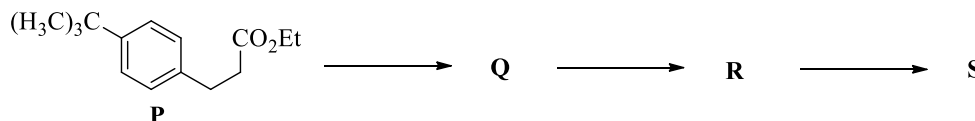
**Space for rough work**

**Answers for the above questions**

<b>Ans for Q.33: (C)</b>	<b>Ans for Q.34: (B)</b>
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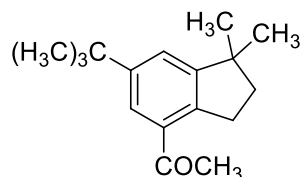
**PARAGRAPH 2**

The reaction of compound **P** with  $\text{CH}_3\text{MgBr}$  (excess) in  $(\text{C}_2\text{H}_5)_2\text{O}$  followed by addition of  $\text{H}_2\text{O}$  gives **Q**. The compound **Q** on treatment with  $\text{H}_2\text{SO}_4$  at  $0^\circ\text{C}$  gives **R**. The reaction of **R** with  $\text{CH}_3\text{COCl}$  in the presence of anhydrous  $\text{AlCl}_3$  in  $\text{CH}_2\text{Cl}_2$  followed by treatment with  $\text{H}_2\text{O}$  produces compound **S**. [Et in compound **P** is ethyl group]

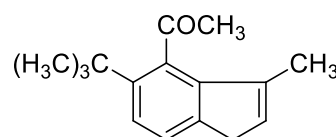


Q.35 The product **S** is

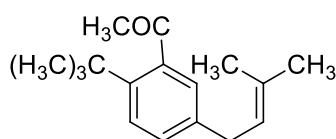
[A]



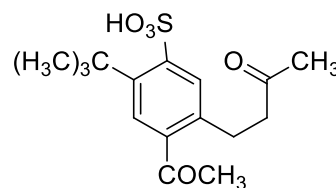
[B]



[C]



[D]



Q.36 The reactions, **Q** to **R** and **R** to **S**, are

- [A] Dehydration and Friedel-Crafts acylation
- [B] Aromatic sulfonation and Friedel-Crafts acylation
- [C] Friedel-Crafts alkylation, dehydration and Friedel-Crafts acylation
- [D] Friedel-Crafts alkylation and Friedel-Crafts acylation

**END OF PART II : CHEMISTRY**

Space for rough work

**Answers for the above questions**

**Ans for Q.35: (A)**

**Ans for Q.36: (D)**

## PART III : MATHEMATICS

### SECTION 1 (Maximum Marks: 21)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened

*Zero Marks* : 0 If none of the bubbles is darkened

*Negative Marks*: -1 In all other cases

Q.37 The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes  $2x + y - 2z = 5$  and  $3x - 6y - 2z = 7$ , is

[A]  $14x + 2y - 15z = 1$

[B]  $14x - 2y + 15z = 27$

[C]  $14x + 2y + 15z = 31$

[D]  $-14x + 2y + 15z = 3$

Q.38 Let  $O$  be the origin and let  $PQR$  be an arbitrary triangle. The point  $S$  is such that

$$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$$

Then the triangle  $PQR$  has  $S$  as its

[A] centroid

[B] circumcentre

[C] incentre

[D] orthocenter

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Space for rough work

### Answers for the above questions

Ans for Q.37: (C)	Ans for Q.38: (D)
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Q.39 If  $y = y(x)$  satisfies the differential equation

$$8\sqrt{x} \left( \sqrt{9 + \sqrt{x}} \right) dy = \left( \sqrt{4 + \sqrt{9 + \sqrt{x}}} \right)^{-1} dx, \quad x > 0$$

and  $y(0) = \sqrt{7}$ , then  $y(256) =$

[A] 3

[B] 9

[C] 16

[D] 80

Q.40 If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is a twice differentiable function such that  $f''(x) > 0$  for all  $x \in \mathbb{R}$ , and  $f\left(\frac{1}{2}\right) = \frac{1}{2}$ ,  $f(1) = 1$ , then

[A]  $f'(1) \leq 0$

[B]  $0 < f'(1) \leq \frac{1}{2}$

[C]  $\frac{1}{2} < f'(1) \leq 1$

[D]  $f'(1) > 1$

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.39: (A)</b>	<b>Ans for Q.40: (D)</b>
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Q.41 How many  $3 \times 3$  matrices  $M$  with entries from  $\{0, 1, 2\}$  are there, for which the sum of the diagonal entries of  $M^T M$  is 5?

- [A] 126                      [B] 198                      [C] 162                      [D] 135

Q.42 Let  $S = \{1, 2, 3, \dots, 9\}$ . For  $k = 1, 2, \dots, 5$ , let  $N_k$  be the number of subsets of  $S$ , each containing five elements out of which exactly  $k$  are odd. Then  $N_1 + N_2 + N_3 + N_4 + N_5 =$

- [A] 210                      [B] 252                      [C] 125                      [D] 126

Q.43 Three randomly chosen nonnegative integers  $x, y$  and  $z$  are found to satisfy the equation  $x + y + z = 10$ . Then the probability that  $z$  is even, is

- [A]  $\frac{36}{55}$                       [B]  $\frac{6}{11}$                       [C]  $\frac{1}{2}$                       [D]  $\frac{5}{11}$

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Space for rough work

### Answers for the above questions

Ans for Q.41: (B)	Ans for Q.42: (D)	Ans for Q.43: (B)
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### SECTION 2 (Maximum Marks: 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened

*Partial Marks* : +1 For darkening a bubble corresponding to **each correct option**, provided NO incorrect option is darkened

*Zero Marks* : 0 If none of the bubbles is darkened

*Negative Marks* : -2 In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.44 If  $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t) dt$ , then

[A]  $g' \left( \frac{\pi}{2} \right) = -2\pi$

[B]  $g' \left( -\frac{\pi}{2} \right) = 2\pi$

[C]  $g' \left( \frac{\pi}{2} \right) = 2\pi$

[D]  $g' \left( -\frac{\pi}{2} \right) = -2\pi$

Q.45 Let  $\alpha$  and  $\beta$  be nonzero real numbers such that  $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$ . Then which of the following is/are true?

[A]  $\tan \left( \frac{\alpha}{2} \right) + \sqrt{3} \tan \left( \frac{\beta}{2} \right) = 0$

[B]  $\sqrt{3} \tan \left( \frac{\alpha}{2} \right) + \tan \left( \frac{\beta}{2} \right) = 0$

[C]  $\tan \left( \frac{\alpha}{2} \right) - \sqrt{3} \tan \left( \frac{\beta}{2} \right) = 0$

[D]  $\sqrt{3} \tan \left( \frac{\alpha}{2} \right) - \tan \left( \frac{\beta}{2} \right) = 0$

### Answers for the above questions

**Q.44: Due to internal review,  
all candidates are awarded  
+4 marks.**

**Q.45: Due to internal review,  
all candidates are awarded  
+4 marks.**

Q.46 If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is a differentiable function such that  $f'(x) > 2f(x)$  for all  $x \in \mathbb{R}$ , and  $f(0) = 1$ , then

- [A]  $f(x)$  is increasing in  $(0, \infty)$                       [B]  $f(x)$  is decreasing in  $(0, \infty)$   
[C]  $f(x) > e^{2x}$  in  $(0, \infty)$                       [D]  $f'(x) < e^{2x}$  in  $(0, \infty)$

Q.47 Let  $f(x) = \frac{1-x(1+|1-x|)}{|1-x|} \cos\left(\frac{1}{1-x}\right)$  for  $x \neq 1$ . Then

- [A]  $\lim_{x \rightarrow 1^-} f(x) = 0$                       [B]  $\lim_{x \rightarrow 1^-} f(x)$  does not exist  
[C]  $\lim_{x \rightarrow 1^+} f(x) = 0$                       [D]  $\lim_{x \rightarrow 1^+} f(x)$  does not exist

Q.48 If  $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$ , then

- [A]  $f'(x) = 0$  at exactly three points in  $(-\pi, \pi)$   
[B]  $f'(x) = 0$  at more than three points in  $(-\pi, \pi)$   
[C]  $f(x)$  attains its maximum at  $x = 0$   
[D]  $f(x)$  attains its minimum at  $x = 0$

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Space for rough work

### Answers for the above questions

Ans for Q.46: (A) and (C)

Ans for Q.47: (A) and (D)

Ans for Q.48: (B) and (C)

Q.49 If the line  $x = \alpha$  divides the area of region  $R = \{(x, y) \in \mathbb{R}^2: x^3 \leq y \leq x, 0 \leq x \leq 1\}$  into two equal parts, then

[A]  $0 < \alpha \leq \frac{1}{2}$

[B]  $\frac{1}{2} < \alpha < 1$

[C]  $2\alpha^4 - 4\alpha^2 + 1 = 0$

[D]  $\alpha^4 + 4\alpha^2 - 1 = 0$

Q.50 If  $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx$ , then

[A]  $I > \log_e 99$

[B]  $I < \log_e 99$

[C]  $I < \frac{49}{50}$

[D]  $I > \frac{49}{50}$

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.49: (B) and (C)</b>	<b>Ans for Q.50: (B) and (D)</b>
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### SECTION 3 (Maximum Marks: 12)

- This section contains **TWO** paragraphs
- Based on each paragraph, there are **TWO** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

*Full Marks* : +3 If only the bubble corresponding to the correct option is darkened  
*Zero Marks* : 0 In all other cases

#### PARAGRAPH 1

Let  $O$  be the origin, and  $\vec{OX}$ ,  $\vec{OY}$ ,  $\vec{OZ}$  be three unit vectors in the directions of the sides  $\vec{QR}$ ,  $\vec{RP}$ ,  $\vec{PQ}$ , respectively, of a triangle  $PQR$ .

Q.51  $|\vec{OX} \times \vec{OY}| =$

- [A]  $\sin(P + Q)$       [B]  $\sin 2R$       [C]  $\sin(P + R)$       [D]  $\sin(Q + R)$

Q.52 If the triangle  $PQR$  varies, then the minimum value of  
 $\cos(P + Q) + \cos(Q + R) + \cos(R + P)$

is

- [A]  $-\frac{5}{3}$       [B]  $-\frac{3}{2}$       [C]  $\frac{3}{2}$       [D]  $\frac{5}{3}$

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Space for rough work

#### Answers for the above questions

Ans for Q.51: (A)

Ans for Q.52: (B)

**PARAGRAPH 2**

Let  $p, q$  be integers and let  $\alpha, \beta$  be the roots of the equation,  $x^2 - x - 1 = 0$ , where  $\alpha \neq \beta$ . For  $n = 0, 1, 2, \dots$ , let  $a_n = p\alpha^n + q\beta^n$ .

**FACT:** If  $a$  and  $b$  are rational numbers and  $a + b\sqrt{5} = 0$ , then  $a = 0 = b$ .

Q.53  $a_{12} =$

- [A]  $a_{11} - a_{10}$       [B]  $a_{11} + a_{10}$       [C]  $2a_{11} + a_{10}$       [D]  $a_{11} + 2a_{10}$

Q.54 If  $a_4 = 28$ , then  $p + 2q =$

- [A] 21      [B] 14      [C] 7      [D] 12

**END OF THE QUESTION PAPER**

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Space for rough work

**Answers for the above questions**

<b>Ans for Q.53: (B)</b>	<b>Ans for Q.54: (D)</b>
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**SPACE FOR ROUGH WORK**